

CAPM, EMPIRICAL CAPM

The Capital Asset Pricing Model (CAPM) is a fundamental paradigm of finance. Simply put, the fundamental idea underlying the CAPM is that risk-averse investors demand higher returns for assuming additional risk, and higher-risk securities are priced to yield higher expected returns than lower-risk securities. The CAPM quantifies the additional return, or risk premium, required for bearing incremental risk. It provides a formal risk-return relationship anchored on the basic idea that only market risk matters, as measured by beta. According to the CAPM, securities are priced such that their:

$$\text{EXPECTED RETURN} = \text{RISK-FREE RATE} + \text{RISK PREMIUM}$$

Denoting the risk-free rate by R_F and the return on the market as a whole by R_M , the CAPM is:

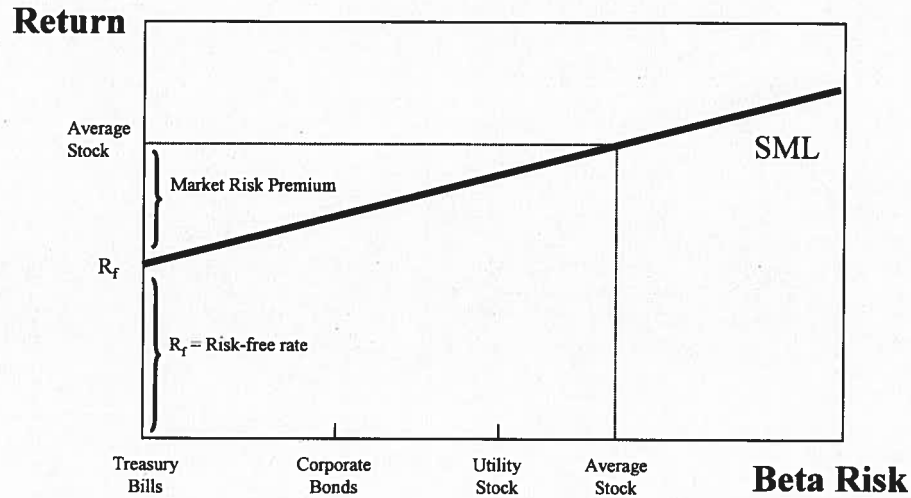
$$K = R_F + \beta(R_M - R_F) \quad (1)$$

Equation 1 is the CAPM expression which asserts that an investor expects to earn a return, K , that could be gained on a risk-free investment, R_F , plus a risk premium for assuming risk, proportional to the security's market risk, also known as beta, β , and the market risk premium, $(R_M - R_F)$, where R_M is the market return. The market risk premium $(R_M - R_F)$ can be abbreviated MRP so that the CAPM becomes:

$$K = R_F + \beta \times \text{MRP} \quad (2)$$

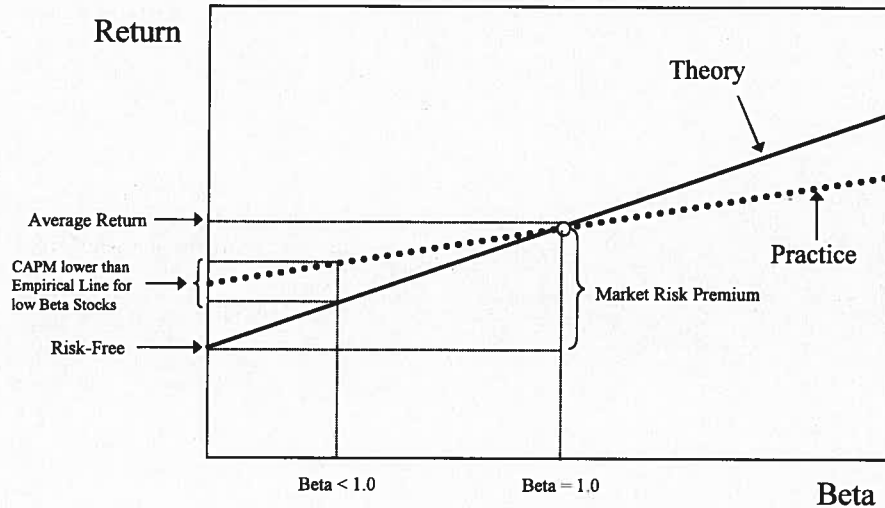
The CAPM risk-return relationship is depicted in the figure below and is typically labeled as the Security Market Line (SML) by the investment community.

CAPM and Risk - Return in Capital Markets



A myriad empirical tests of the CAPM have shown that the risk-return tradeoff is not as steeply sloped as that predicted by the CAPM, however. That is, low-beta securities earn returns somewhat higher than the CAPM would predict, and high-beta securities earn less than predicted. In other words, the CAPM tends to overstate the actual sensitivity of the cost of capital to beta: low-beta stocks tend to have higher returns and high-beta stocks tend to have lower risk returns than predicted by the CAPM. The difference between the CAPM and the type of relationship observed in the empirical studies is depicted in the figure below. This is one of the most widely known empirical findings of the finance literature. This extensive literature is summarized in Chapter 13 of Dr. Morin's book [Regulatory Finance, Public Utilities Report Inc., Arlington, VA, 1994].

Risk vs Return Theory vs. Practice



A number of refinements and expanded versions of the original CAPM theory have been proposed to explain the empirical findings. These revised CAPMs typically produce a risk-return relationship that is flatter than the standard CAPM prediction. The following equation makes use of these empirical findings by flattening the slope of the risk-return relationship and increasing the intercept:

$$K = R_F + \alpha + \beta (MRP - \alpha) \quad (3)$$

where α is the "alpha" of the risk-return line, a constant determined empirically, and the other symbols are defined as before. Alternatively, Equation 3 can be written as follows:

$$K = R_F + a MRP + (1-a) \beta MRP \quad (4)$$

where a is a fraction to be determined empirically. Comparing Equations 3 and 4, it is easy to see that alpha equals 'a' times MRP, that is, $\alpha = a \times M R P$

Theoretical Underpinnings

The obvious question becomes what would produce a risk return relationship which is flatter than the CAPM prediction, or in other words, how do you explain the presence of "alpha" in the above equation. The exclusion of variables aside from beta would produce this result. Three such variables are noteworthy: dividend yield, skewness, and hedging potential.

The dividend yield effects stem from the differential taxation on corporate dividends and capital gains. The standard CAPM does not consider the regularity of dividends received by investors. Utilities generally maintain high dividend payout ratios relative to the market, and by ignoring dividend yield, the CAPM provides biased cost of capital estimates. To the extent that dividend income is taxed at a higher rate than capital gains, investors will require higher pre-tax returns in order to equalize the after-tax returns provided by high-yielding stocks (e.g. utility stocks) with those of low-yielding stocks. In other words, high-yielding stocks must offer investors higher pre-tax returns. Even if dividends and capital gains are undifferentiated for tax purposes, there is still a tax bias in favor of earnings retention (lower dividend payout), as capital gains taxes are paid only when gains are realized.

Empirical studies by Litzenberger and Ramaswamy (1979), Litzenberger et al. (1980) and Rosenberg and Marathe (1975) find that security returns are positively related to dividend yield as well as to beta. These results are consistent with after-tax extensions of the CAPM developed by Brennan (1973) and Litzenberger and Ramaswamy (1979) and suggest that the relationship between return, beta, and dividend yield should be estimated and employed to calculate the cost of equity capital.

As far as skewness is concerned, investors are more concerned with losing money than with total variability of return. If risk is defined as the probability of loss, it appears more logical to measure risk as the probability of achieving a return which is below the expected return. The traditional CAPM provides downward-biased estimates of cost of capital to the extent that these skewness effects are significant. As shown by Kraus and Litzenberger (1976), expected return depends on both on a stock's systematic risk (beta) and the systematic skewness. Empirical studies by Kraus and Litzenberger (1976),

Friend, Westerfield, and Granito (1978), and Morin (1981) found that, in addition to beta, skewness of returns has a significant negative relationship with security returns. This result is consistent with the skewness version of the CAPM developed by Rubinstein (1973) and Kraus and Litzenberger (1976).

This is particularly relevant for public utilities whose future profitability is constrained by the regulatory process on the upside and relatively unconstrained on the downside in the face of socio-political realities of public utility regulation. The process of regulation, by restricting the upward potential for returns and responding sluggishly on the downward side, may impart some asymmetry to the distribution of returns, and is more likely to result in utilities earning less, rather than more, than their cost of capital. The traditional CAPM provides downward-biased estimates of cost of capital to the extent that these skewness effects are significant.

As far as hedging potential is concerned, investors are exposed to another kind of risk, namely, the risk of unfavorable shifts in the investment opportunity set. Merton (1973) shows that investors will hold portfolios consisting of three funds: the risk-free asset, the market portfolio, and a portfolio whose returns are perfectly negatively correlated with the riskless asset so as to hedge against unforeseen changes in the future risk-free rate. The higher the degree of protection offered by an asset against unforeseen changes in interest rates, the lower the required return, and conversely. Merton argues that low beta assets, like utility stocks, offer little protection against changes in interest rates, and require higher returns than suggested by the standard CAPM.

Another explanation for the CAPM's inability to fully explain the process determining security returns involves the use of an inadequate or incomplete market index. Empirical studies to validate the CAPM invariably rely on some stock market index as a proxy for the true market portfolio. The exclusion of several asset categories from the definition of market index mis-specifies the CAPM and biases the results found using only stock market data. Kolbe and Read (1983) illustrate the biases in beta estimates which result from applying the CAPM to public utilities. Unfortunately, no comprehensive and easily accessible data exist for several classes of assets, such as mortgages and business investments, so that the exact relation between return and stock betas predicted by the CAPM does not exist. This suggests that the empirical relationship

between returns and stock betas is best estimated empirically (ECAPM) rather than by relying on theoretical and elegant CAPM models expanded to include missing assets effects. In any event, stock betas may be highly correlated with the true beta measured with the true market index.

Yet another explanation for the CAPM's inability to fully explain the observed risk-return tradeoff involves the possibility of constraints on investor borrowing that run counter to the assumptions of the CAPM. In response to this inadequacy, several versions of the CAPM have been developed by researchers. One of these versions is the so-called zero-beta, or two-factor, CAPM which provides for a risk-free return in a market where borrowing and lending rates are divergent. If borrowing rates and lending rates differ, or there is no risk-free borrowing or lending, or there is risk-free lending but no risk-free borrowing, then the CAPM has the following form:

$$K = R_z + \beta(R_m - R_f)$$

The model, christened the zero-beta model, is analogous to the standard CAPM, but with the return on a minimum risk portfolio which is unrelated to market returns, R_z , replacing the risk-free rate, R_f . The model has been empirically tested by Black, Jensen, and Scholes (1972), who found a flatter than predicted CAPM, consistent with the model and other researchers' findings.

The zero-beta CAPM cannot be literally employed in cost of capital projections, since the zero-beta portfolio is a statistical construct difficult to replicate.

Empirical Evidence

A summary of the empirical evidence on the magnitude of alpha is provided in the table below.

Empirical Evidence on the Alpha Factor		
Author	Range of alpha	Period relied upon
Fischer (1993)	-3.6% to 3.6%	1931-1991
Fischer, Jensen and Scholes (1972)	-9.61% to 12.24%	1931-1965
Fama and McBeth (1972)	4.08% to 9.36%	1935-1968
Fama and French (1992)	10.08% to 13.56%	1941-1990
Litzenberger and Ramaswamy (1979)	5.32% to 8.17%	
Litzenberger, Ramaswamy and Sosin (1980)	1.63% to 5.04%	1926-1978
Pettengill, Sundaram and Mathur (1995)	4.6%	
Morin (1994)	2.0%	1926-1984
Harris, Marston, Mishra, and O'Brien	2.0%	1983-1998

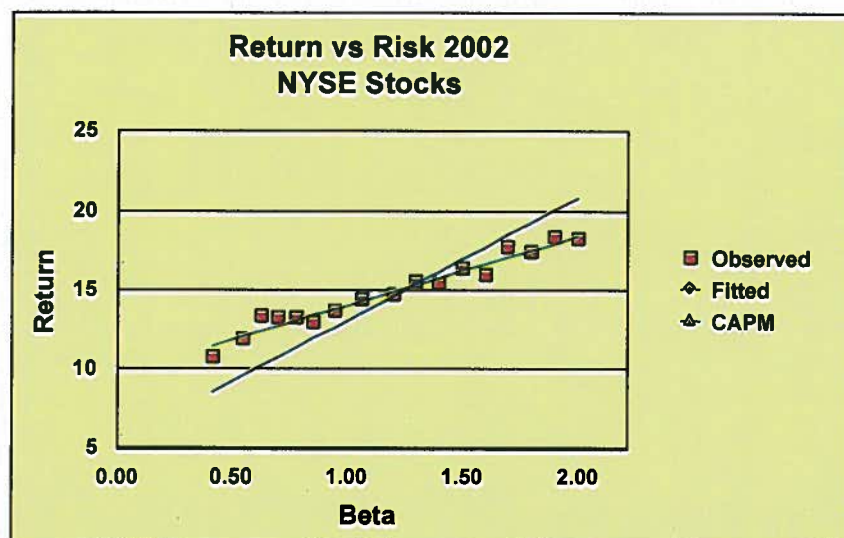
Given the observed magnitude of alpha, the empirical evidence indicates that the risk-return relationship is flatter than that predicted by the CAPM. Typical of the empirical evidence is the findings cited in Morin (1994) over the period 1926-1984 indicating that the observed expected return on a security is related to its risk by the following equation:

$$K = .0829 + .0520 \beta$$

Given that the risk-free rate over the estimation period was approximately 6%, this relationship implies that the intercept of the risk-return relationship is higher than the 6% risk-free rate, contrary to the CAPM's prediction. Given that the average return on an average risk stock exceeded the risk-free rate by about 8.0% in that period, that is, the market risk premium ($R_M - R_F$) = 8%, the intercept of the observed relationship between return and beta exceeds the risk-free rate by about 2%, suggesting an alpha factor of 2%.

Most of the empirical studies cited in the above table utilize raw betas rather than Value Line adjusted betas because the latter were not available over most of the time periods covered in these studies. A study of the relationship between return and adjusted beta is reported on Table 6-7 in Ibbotson Associates Valuation Yearbook 2001. If we exclude the portfolio of very small cap stocks from the relationship due to significant size effects, the relationship between the arithmetic mean return and beta for the remaining portfolios is flatter than predicted and the intercept slightly higher than predicted by the CAPM, as shown on the graph below. It is noteworthy that the Ibbotson study relies on adjusted betas as stated on page 95 of the aforementioned study.

CAPM vs ECAPM

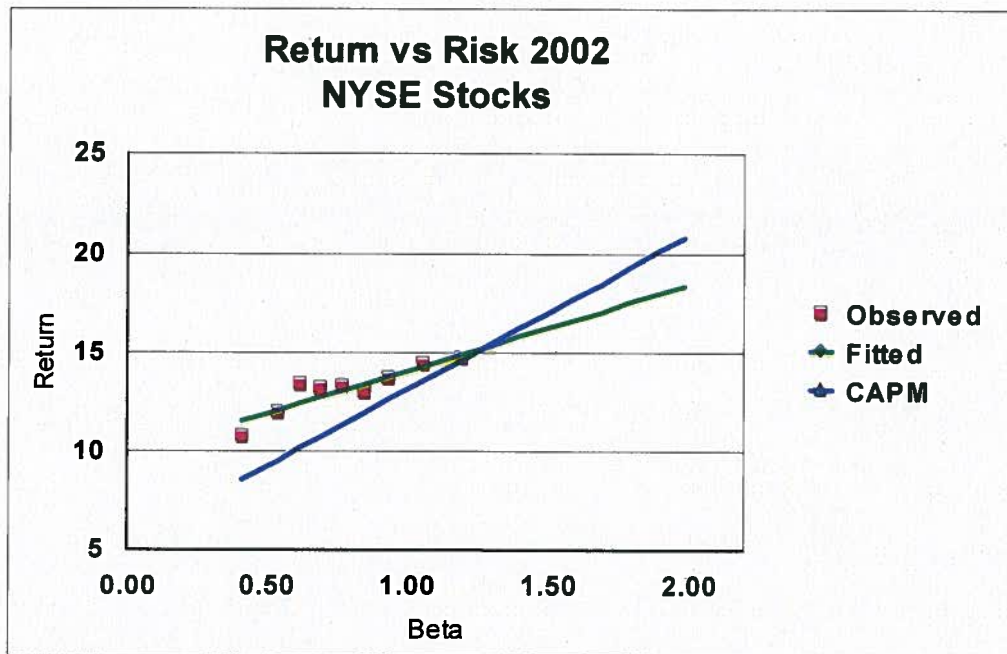


Another study by Morin in May 2002 provides empirical support for the ECAPM. All the stocks covered in the Value Line Investment Survey for Windows for which betas and returns data were available were retained for analysis. There were nearly 2000 such stocks. The expected return was measured as the total shareholder return ("TSR") reported by Value Line over the past ten years. The Value Line adjusted beta was also retrieved from the same data base. The nearly 2000 companies for which all data were available were ranked in ascending order of beta, from lowest to highest. In order to palliate measurement error, the nearly 2000 securities were grouped into ten portfolios of

approximately 180 securities for each portfolio. The average returns and betas for each portfolio were as follows:

Portfolio #	Beta	Return
portfolio 1	0.41	10.87
portfolio 2	0.54	12.02
portfolio 3	0.62	13.50
portfolio 4	0.69	13.30
portfolio 5	0.77	13.39
portfolio 6	0.85	13.07
portfolio 7	0.94	13.75
portfolio 8	1.06	14.53
portfolio 9	1.19	14.78
portfolio 10	1.48	20.78

It is clear from the graph below that the observed relationship between DCF returns and Value Line adjusted betas is flatter than that predicted by the plain vanilla CAPM. The observed intercept is higher than the prevailing risk-free rate of 5.7% while the slope is less than equal to the market risk premium of 7.7% predicted by the plain vanilla CAPM for that period.



In an article published in Financial Management, Harris, Marston, Mishra, and O'Brien ("HMMO") estimate ex ante expected returns for S&P 500 companies over the period 1983-1998¹. HMMO measure the expected rate of return (cost of equity) of each dividend-paying stock in the S&P 500 for each month from January 1983 to August 1998 by using the constant growth DCF model. They then investigate the relation between the risk premium (expected return over the 20-year Treasury bond yield) estimates for each month to equity betas as of that same month (5-year raw betas).

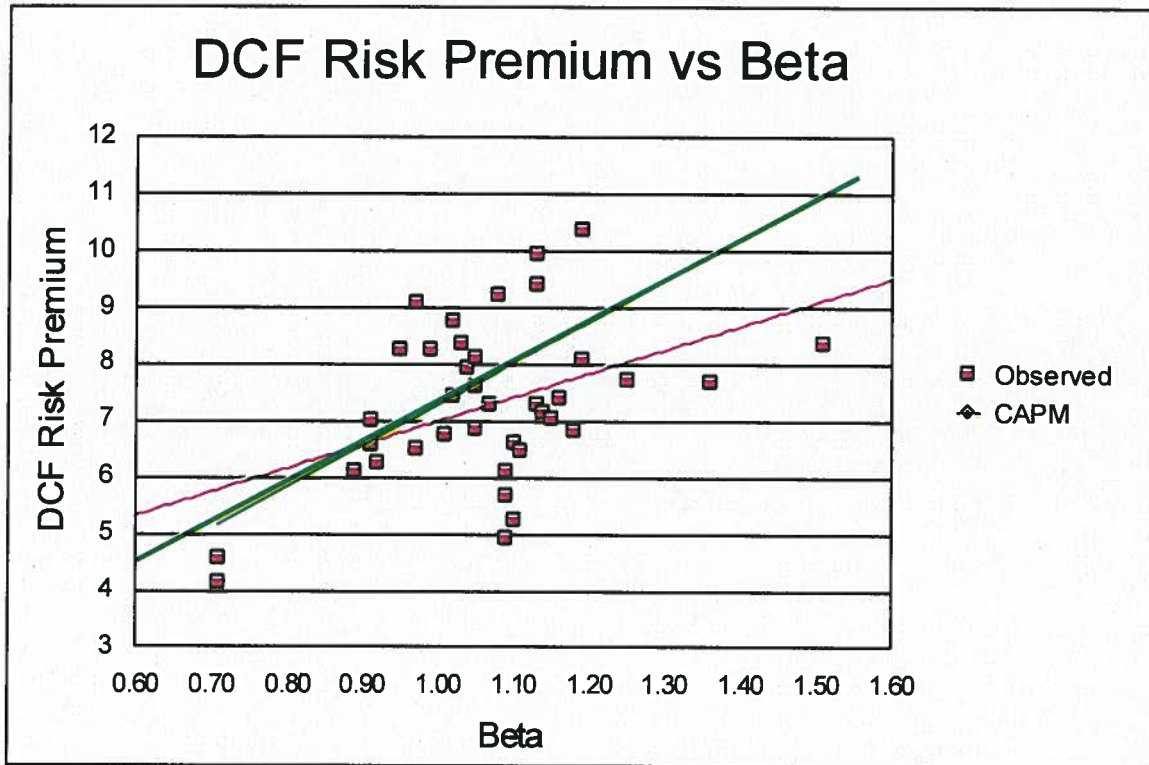
The table below, drawn from HMMO Table 4, displays the average estimate prospective risk premium (Column 2) by industry and the corresponding beta estimate for that industry, both in raw form (Column 3) and adjusted form (Column 4). The latter were calculated with the traditional Value Line – Merrill Lynch – Bloomberg adjustment methodology by giving 1/3 weight of to a beta estimate of 1.00 and 2/3 weight to the raw beta estimate.

¹ Harris, R. S., Marston, F. C., Mishra, D. R., and O'Brien, T. J., "Ex Ante Cost of Equity Estimates of S&P 500 Firms: The Choice Between Global and Domestic CAPM," Financial

Table A-1 Risk Premium and Beta Estimates by Industry

	Industry	DCF Risk Premium	Raw Industry Beta	Adjusted Industry Beta
	(1)	(2)	(3)	(4)
1	Aero	6.63	1.15	1.10
2	Autos	5.29	1.15	1.10
3	Banks	7.16	1.21	1.14
4	Beer	6.60	0.87	0.91
5	BldMat	6.84	1.27	1.18
6	Books	7.64	1.07	1.05
7	Boxes	8.39	1.04	1.03
8	BusSv	8.15	1.07	1.05
9	Chems	6.49	1.16	1.11
10	Chips	8.11	1.28	1.19
11	Clths	7.74	1.37	1.25
12	Cnstr	7.70	1.54	1.36
13	Comps	9.42	1.19	1.13
14	Drugs	8.29	0.99	0.99
15	ElcEq	6.89	1.08	1.05
16	Energy	6.29	0.88	0.92
17	Fin	8.38	1.76	1.51
18	Food	7.02	0.86	0.91
19	Fun	9.98	1.19	1.13
20	Gold	4.59	0.57	0.71
21	Hlth	10.40	1.29	1.19
22	Hsld	6.77	1.02	1.01
23	Insur	7.46	1.03	1.02
24	LabEq	7.31	1.10	1.07
25	Mach	7.32	1.20	1.13
26	Meals	7.98	1.06	1.04
27	MedEq	8.80	1.03	1.02
28	Pap	6.14	1.13	1.09
29	PerSv	9.12	0.95	0.97
30	Retail	9.27	1.12	1.08
31	Rubber	7.06	1.22	1.15
32	Ships	1.95	0.95	0.97
33	Stee	4.96	1.13	1.09
34	Telc	6.12	0.83	0.89
35	Toys	7.42	1.24	1.16
36	Trans	5.70	1.14	1.09
37	Txtls	6.52	0.95	0.97
38	Util	4.15	0.57	0.71
39	Whsl	8.29	0.92	0.95
	MEAN	7.19		

The observed statistical relationship between expected return and **adjusted beta** is shown in the graph below along with the CAPM prediction:



If the plain vanilla version of the CAPM is correct, then the intercept of the graph should be zero, recalling that the vertical axis represents returns in excess of the risk-free rate. Instead, the observed intercept is approximately 2%, that is approximately equal to 25% of the expected market risk premium of 7.2% shown at the bottom of Column 2 over the 1983-1998 period, as predicted by the ECAPM. The same is true for the slope of the graph. If the plain vanilla version of the CAPM is correct, then the slope of the relationship should equal the market risk premium of 7.2%. Instead, the observed slope of close to 5% is approximately equal to 75% of the expected market risk premium of 7.2%, as predicted by the ECAPM.

In short, the HMMO empirical findings are quite consistent with the predictions of the ECAPM.

Practical Implementation of the ECAPM

The empirical evidence reviewed above suggests that the expected return on a security is related to its risk by the following relationship:

$$K = R_F + \alpha + \beta (MRP - \alpha) \quad (5)$$

or, alternatively by the following equivalent relationship:

$$K = R_F + a MRP + (1-a) \beta MRP \quad (6)$$

The empirical findings support values of α from approximately 2% to 7%. If one is using the short-term U.S. Treasury Bills yield as a proxy for the risk-free rate, and given that utility stocks have lower than average betas, an alpha in the lower range of the empirical findings, 2% - 3% is reasonable, albeit conservative.

Using the long-term U.S. Treasury yield as a proxy for the risk-free rate, a lower alpha adjustment is indicated. This is because the use of the long-term U.S. Treasury yield as a proxy for the risk-free rate partially incorporates the desired effect of using the ECAPM². An alpha in the range of 1% - 2% is therefore reasonable.

To illustrate, consider a utility with a beta of 0.80. The risk-free rate is 5%, the MRP is 7%, and the alpha factor is 2%. The cost of capital is determined as follows:

$$\begin{aligned} K &= R_F + \alpha + \beta (MRP - \alpha) \\ K &= 5\% + 2\% + 0.80(7\% - 2\%) \\ &= 11\% \end{aligned}$$

A practical alternative is to rely on the second variation of the ECAPM:

$$K = R_F + a MRP + (1-a) \beta MRP$$

² The Security Market Line (SML) using the long-term risk-free rate has a higher intercept and a flatter slope than the SML using the short-term risk-free rate

With an alpha of 2%, a MRP in the 6% - 8% range, the 'a' coefficient is 0.25, and the ECAPM becomes³:

$$K = R_F + 0.25 \text{ MRP} + 0.75 \beta \text{ MRP}$$

Returning to the numerical example, the utility's cost of capital is:

$$\begin{aligned} K &= 5\% + 0.25 \times 7\% + 0.75 \times 0.80 \times 7\% \\ &= 11\% \end{aligned}$$

For reasonable values of beta and the MRP, both renditions of the ECAPM produce results that are virtually identical⁴.

³ Recall that alpha equals 'a' times MRP, that is, $\alpha = a \text{ MRP}$, and therefore $a = \alpha/\text{MRP}$. If alpha is 2%, then $a = 0.25$

⁴ In the Morin (1994) study, the value of "a" was actually derived by systematically varying the constant "a" in equation 6 from 0 to 1 in steps of 0.05 and choosing that value of 'a' that minimized the mean square error between the observed relationship between return and beta:

$$K = 0.0829 + .0520 \beta$$

The value of a that best explained the observed relationship was 0.25.

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