VEDO EXHIBIT NO. 5.0

BEFORE
THE PUBLIC UTILITIES COMMISSION OF OHIO

In the Matter of the Application of Vectren Energy Delivery of Ohio, Inc., for Approval of an Increase in Gas Rate	)))	Case No. 18-0298-GA-AIR
In the Matter of the Application of Vectren Energy Delivery of Ohio, Inc., for Approval of an Alternative Rate Plan	)))	Case No. 18-0299-GA-ALT

DIRECT TESTIMONY OF
MICHAEL J. VILBERT
ON BEHALF OF
VECTREN ENERGY DELIVERY OF OHIO, INC.

_____	Management policies, practices, and organization
_____	Operating income
_____	Rate base
_____	Allocations
X	Rate of return (Cost of Common Equity Capital)
_____	Rates and tariffs
_____	Other

TABLE OF CONTENTS

Direct Testimony of
Michael J. Vilbert

I. INTRODUCTION AND SUMMARY

Q1. Please state your name and address for the record.

A1. My name is Michael J. Vilbert. My business address is The Brattle Group, 201 Mission Street, Suite 2800, San Francisco, CA 94105, USA.

Q2. Please summarize your background and experience.

A2. I am a Principal Emeritus of The Brattle Group ("Brattle"), an economic, environmental and management consulting firm with offices in Boston, Washington, London, San Francisco, Madrid, Rome, Toronto, and New York City. My work concentrates on financial and regulatory economics. I hold a B.S. from the U.S. Air Force Academy and a Ph.D. in finance from the Wharton School of Business at the University of Pennsylvania. Appendix A provides more detail on my qualifications.

Q3. What is the purpose of your testimony in this proceeding?

A3. I have been asked by Vectren Energy Delivery of Ohio, Inc. ("Vectren" or the "Company") to estimate the cost of capital for the Company. Specifically, I provide return on equity ("ROE") estimates derived from a sample of comparable risk, regulated gas local distribution utility companies ("gas LDCs"). I also consider the financial risk of the Company's capital structure ratio as of December 31, 2017 to arrive at my recommendation for the allowed ROE.

Q4. Are you sponsoring any exhibits?

A4. Yes, I am sponsoring Attachment A which includes the following schedules:

Attachment	Schedule	Description
A	D5	Cost of Common Shareholders' Equity
A	D5.1	Table of Contents
A	D5.2	Classification of Companies by Assets

A	D5.3	Market Value of the Expanded Sample
A	D5.4	Capital Structure Summary of the Expanded Sample
A	D5.5	Estimated Growth Rates of the Expanded Sample
A	D5.6	DCF Cost of Equity of the Expanded Sample
A	D5.7	Overall After-Tax DCF Cost of Capital of the Expanded Sample
A	D5.8	DCF Cost of Equity at Vectren's Capital Structure
A	D5.9	Risk-Free Rates
A	D5.10	Risk Positioning Cost of Equity of the Expanded Sample
A	D5.11	Overall After-Tax Risk Positioning Cost of Capital of the Expanded Sample
A	D5.12	Risk Positioning Cost of Equity at Vectren's Capital Structure
A	D5.13	Hamada Adjustment to Obtain Unlevered Asset Beta
A	D5.14	Expanded Sample Average Asset Beta Relevered at Vectren's Capital Structure
A	D5.15	Risk-Positioning Cost of Equity using Hamada-Adjusted Betas
A	D5.16	Risk Premiums Determined by Relationship Between Authorized ROEs and Long-term Treasury Bond Rates
A	D5.17	Academic Literature on Financial Risk Adjustments

Q5. Were these exhibits and schedules prepared by you or under your direction?

A5. Yes.

Q6. Can you summarize the parts of your background and experience that are particularly relevant to your testimony on these matters?

A6. Brattle's specialties include financial economics, regulatory economics, and the gas, water, and electric industries. I have worked in the areas of cost of capital, investment risk, and related matters for many industries, regulated and unregulated alike, in many forums. A partial list of the regulators before which I have testified or filed cost of capital testimony include the Arizona Corporation Commission, the Pennsylvania Public Utility Commission, the Public Service Commission of West

Virginia, the Tennessee Regulatory Authority, the Public Service Commission of Wisconsin, the South Dakota Utilities Commission, the California Public Utilities Commission, and the Federal Energy Regulatory Commission ("FERC"). I have also testified in Canada before the Canadian National Energy Board, the Alberta Energy and Utilities Board, the Ontario Energy Board, the Quebec Régie de l'énergie, and the Labrador & Newfoundland Board of Commissioners of Public Utilities. I have testified previously before the Public Utilities Commission ("Commission") of Ohio. Appendix A contains more information on my professional qualifications.

Q7. What are the steps in your analysis?

A7. To estimate the Company's cost of capital, I analyzed a sample of gas LDCs, identified as being similar in risk and business operations to Vectren, specifically the regulated gas local distribution business. I estimate the ROE for each sample company using both the risk positioning and the discounted cash flow ("DCF") approaches. The risk positioning approach consists of analyses based upon the Capital Asset Pricing Model ("CAPM") and the Empirical CAPM ("ECAPM"). The ROE estimates from both models are then combined with market value capital structure information and the market costs of debt and preferred stock for each sample company to compute each firm's overall cost of capital, i.e., its after-tax weighted-average cost of capital ("ATWACC"). I also provide an ROE estimate based upon the risk premium model.

Q8. What is the result of the cost of capital estimation process?

A8. The result of this process is a sample average ATWACC for each cost of equity estimation method. I then report the cost of equity consistent with the sample's average estimated ATWACC as if the sample's average market-value capital structure had been one with a 50.6 percent equity ratio, which was Vectren's equity ratio as of December 31, 2017. This procedure results in a ROE that is consistent with both the financial risk inherent in the Company's capital structure and the market-determined information on the sample's average overall cost of capital.

Q9. Do you present any other methods to take differences in financial risk into account?

A9. Yes. Other than the ATWACC method, I use the method originally proposed by Professor Robert S. Hamada to account for the differences in financial risk through adjustments to the beta estimate for a firm.[1] This procedure is common amongst finance practitioners and well-established in academic literature. I present this method, which I refer to as the Hamada adjustment procedures, for the risk positioning analyses alongside the ATWACC method in order to further inform my recommendations that account for differences in the financial risk between the companies in my sample and Vectren.

Q10. How does the ongoing uncertainty in the financial markets affect the cost of capital for a regulated utility?

A10. The cost of capital is higher than a mechanical implementation of the ROE estimation models may suggest. Although economic conditions have improved substantially since the start of the crisis in about mid-2008, uncertainty remains in the capital markets due, in part, to the disappointing rate of economic growth, not only in the U.S., but also worldwide. Worries about the low interest rate outlook in Europe and Japan as well as the United Kingdom's exit from the European Union have added to the concern. In addition, long-term government bond yields, which had dropped dramatically after the 2008-2009 credit crisis to unusually low levels, remain depressed relative to both historical levels and forecasts of future interest rates. The increased volatility in the stock market at the beginning of February 2018 demonstrates that substantial uncertainty remains in the capital markets.

As a result, bond yield spreads remain higher than before the credit crisis,[2] both for riskier assets as well as for less risky investments such as investment grade-rated

---

[1] Hamada, R.S., "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stock," *The Journal of Finance*, 27(2), 1971, pp. 435-452. See Attachment A, Schedule No. D5-17 at 56-74.

[2] The yield spread in this case is the difference between the yield on a risky corporate debt security and the yield on U.S. Treasury debt of comparable maturity.

utility debt, as illustrated in Table 1 below. Although the capital market indices have returned to and have now exceeded their pre-crisis levels, the recovery remains fragile in part because of the weakness in parts of the rest of the world. I discuss economic conditions and the effect of the credit crisis on the cost of capital and its various components, including the long-term risk-free interest rate, in more detail in Section III below.

This uncertainty in the financial markets also affects the results of the estimation models, because both the risk positioning model and the DCF model are based upon the assumption that economic conditions are stable. That assumption is not currently met, so estimating the cost of capital under current conditions is more complicated than it would normally be.

Q11. Do you adjust your analyses to account for the remaining market uncertainty?

A11. Yes. Because the uncertainty in financial markets affects the cost of capital for all companies, including regulated utilities such as Vectren, I modified the parameters of the risk positioning model to recognize the effect of the increased volatility in the capital markets as well as the overall decline in long-term risk-free interest rates on the cost of capital. Specifically, I analyzed scenarios using two different estimates of the market risk premium ("MRP") and risk-free interest rate for use in the risk positioning model. These scenarios are discussed in more detail below. Further, given the current economic uncertainty and the downward bias it creates in the CAPM model results, I also place substantial weight on the results of the DCF analyses in determining the range of reasonableness for the ROE, for reasons explained later in this testimony.

Q12. Can you summarize your findings about the expanded sample's costs of capital?

A12. The sample ROE estimates range from a low of 9.1 percent to a high of 13.7 percent, but I believe that the estimates at the lower end of the range are not reliable because they do not fully consider the effect of the ongoing uncertainty in the financial markets and the downward pressure on the risk-free interest rate. Conversely, the estimates at the upper end of the range reflect the adjustment for the ongoing uncertainty in the capital market and are more reliable. For a regulated natural gas LDC of average business risk and with an equity ratio consistent with Vectren's equity ratio of approximately 50.6 percent, the best estimate of the range for the cost of equity is from 10 percent to 11 percent.

Q13. What ROE do you recommend for the Company in this proceeding?

A13. I recommend that the Company be allowed an ROE of 10¾ percent on the equity financed portion of its rate base.[3] This is above the midpoint of the range of 10 percent to 11 percent that I believe is reasonable for the sample companies comparable to Vectren's financial and business risk because I believe that Vectren is of somewhat greater risk than the average company in the sample. In addition, the current market uncertainty associated with new tariffs and the effect of the recent reductions in corporate income tax rates have increased risks for regulated utilities beyond what a mechanical review of the historical record would indicate. Moreover, the rating agencies have recognized that the new tax law puts pressure on regulated companies' credit metrics which is an additional factor to consider when determining the allowed ROE for Vectren.[4]

Q14. How is your testimony organized?

A14. Section II formally defines the cost of capital and touches on the principles relating to estimating the cost of capital and the effect of capital structure on the cost of equity. Section III discusses the current capital market conditions and the effect of income

---

[3] I report my recommended ROE to the nearest ¼ percentage point because I do not believe that the cost of capital can be estimated more precisely than that even though the model results can be reported to several decimal places.

[4] "Moody's changes outlooks on 25 US regulated utilities primarily impacted by tax reform," Moody's Investor Service, Global Credit Research, January 19, 2018, and "Tax reform is credit negative for sector, but impact varies by company," Moody's Investor Service, Sector Comment, January 24, 2018. Also "U.S. Tax Reform: For Utilities' Credit Quality, Challenges Abound," S&P Global Ratings, Rating Direct, January 24, 2018; and "Tax Reform Impact on the U.S. Utilities, Power & Gas Sector: Tax Reform Creates Near-Term Credit Pressure for Regulated Utilities and Holding Companies," Fitch Ratings, Special Report, January 24, 2018.

tax reform on the cost of capital. Section IV discusses the selection of the expanded sample, and Section V presents the methods used to estimate the cost of capital for the sample; provides the associated numerical analyses; and explains the basis of my conclusions for the sample's overall costs of capital. Section VI concludes my testimony. The calculations supporting my analyses are provided in Exhibit No. D.5.

II. COST OF CAPITAL THEORY

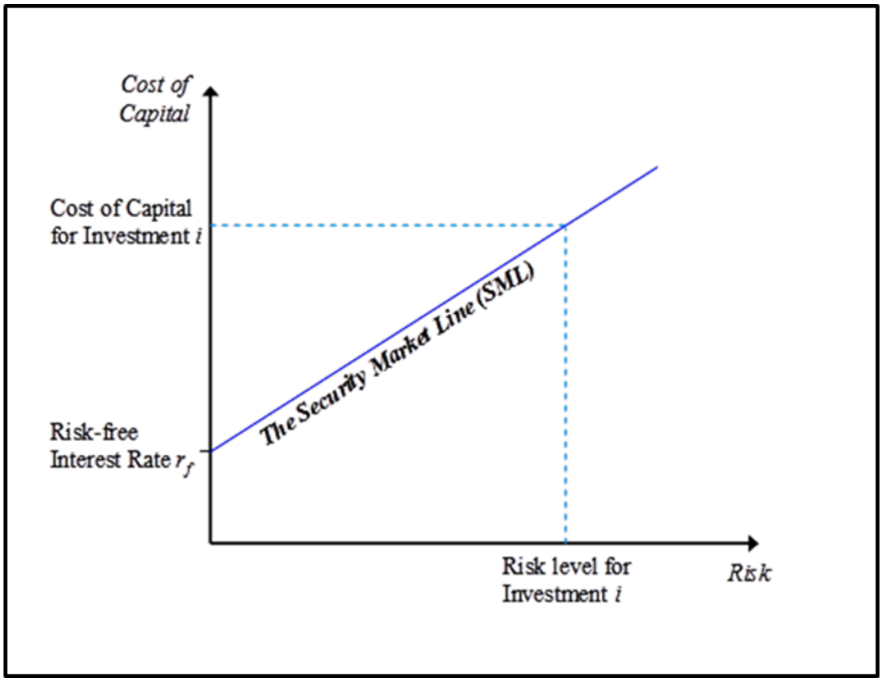
A. COST OF CAPITAL AND RISK

Q15. How is the "cost of capital" formally defined?

A15. The cost of capital is defined as the expected rate of return in capital markets on alternative investments of equivalent risk. In other words, it is the rate of return investors require based on the risk-return alternatives available in competitive capital markets. The cost of capital is a type of opportunity cost: it represents the rate of return that investors could expect to earn elsewhere without bearing more risk. "Expected" is used in the statistical sense: the mean of the distribution of possible outcomes. The terms "expect" and "expected," as in the definition of the cost of capital itself, refer to the probability-weighted average over all possible outcomes.

The definition of the cost of capital recognizes a tradeoff between risk and return that can be represented by the "security market risk-return line" or "Security Market Line" for short. This line is depicted in Figure 1. The higher the risk, the higher the cost of capital required.

Figure 1
The Security Market Line



Q16. Why is the cost of capital relevant in rate regulation?

A16. It has become routine in U.S. rate regulation to accept the "cost of capital" as the right expected rate of return on utility investments.[5] That practice is viewed as consistent with the U.S. Supreme Court's opinions in Bluefield Water Works & Improvement Co. v. Public Service Commission of West Virginia, 262 U.S. 679 (1923), and Federal Power Commission v. Hope Natural Gas Co., 320 U.S. 591 (1944).

From an economic perspective, rate levels that give investors a fair opportunity to earn the cost of capital are the lowest levels that compensate investors for the risks they bear. Over the long run, an expected return above the cost of capital makes customers overpay for service. Regulatory commissions normally try to prevent such outcomes unless there are offsetting benefits (e.g., from incentive regulation that reduces future costs). At the same time, an expected return below the cost of capital

[5] A formal link between the cost of capital as defined by financial economics and the right expected rate of return for utilities is set forth by Stewart C. Myers, *Application of Finance Theory to Public Utility Rate Cases, Bell Journal of Economics & Management Science* 3:58-97 (1972).

does a disservice not just to investors but, importantly, to customers as well. Such a return denies the company the ability to attract capital, to maintain its financial integrity, and to expect a return commensurate with that of other enterprises attended by corresponding risks and uncertainties.

More important for customers, however, are the broader economic consequences of providing an inadequate return to the company's investors. In the short run, deviations from the expected rate of return on the rate base from the cost of capital may seemingly create a "zero-sum game"—investors gain if customers are overcharged, and customers gain if investors are shortchanged. But in fact, in the short run, such actions may adversely affect the utility's ability to provide stable and favorable rates because some potential efficiency investments may be delayed or because the company is forced to file more frequent rate cases. Moreover, in the long run, inadequate returns are likely to cost customers—and society generally—far more than may be saved in the short run. Inadequate returns lead to inadequate investment, whether for maintenance or for new plant and equipment. Without access to investor capital, the company may be forced to forgo opportunities to maintain, upgrade, and expand its systems and facilities in ways that decrease long run costs. Indeed, the cost to consumers of an undercapitalized industry can be far greater than any short-run gains from shortfalls in the cost of capital. This is especially true in capital-intensive industries (such as the natural gas distribution industry), which feature systems that take a long time to decay. Such long-lived infrastructure assets cannot be repaired or replaced overnight, because of the time necessary to plan and construct the facilities. Thus, it is in the customers' interest not only to make sure the return investors expect does not exceed the cost of capital, but also to make sure that the return does not fall short of the cost of capital. In fact, research has shown that there is a positive correlation between allowed ROEs from the regulators and customer satisfaction ratings.[6] In other words, the customers of utilities in more supportive regulatory environments have higher satisfaction in the quality of service.

[6] Barclay's Research, "North America Power & Utilities: March Preview/February Review," February 17, 2017.

Of course, the cost of capital cannot be estimated with perfect certainty, and other aspects of the way the revenue requirement is set may mean investors expect to earn more or less than the cost of capital, even if the allowed rate of return equals the cost of capital exactly. However, a commission that sets rates so investors expect to earn the cost of capital on average treats both customers and investors fairly, and acts in the long-run interests of both groups.

B. Relationship Between Capital Structure and the Cost of Equity

Q17. What did you mean by the "ATWACC" mentioned earlier?

A17. The ATWACC is calculated as the weighted average of the after-tax cost of debt capital and the cost of equity. Specifically, the following equation pertains:[7]

$$ATWACC = r_D \times (1 - T_c) \times \%D + r_E \times \%E \qquad (1)$$

where r_D = market cost of debt,

r_E = market cost of equity,

T_C = corporate income tax rate,

$\%D$ = percent debt in the capital structure, and

$\%E$ = percent equity in the capital structure

The ATWACC is commonly referred to as the WACC in financial textbooks and is used in investment decisions.[8] The return on equity consistent with the sample's overall cost of capital estimate (the ATWACC), the market cost of debt, the corporate income tax rate, and the amount of debt and common equity in the capital structure can be determined by solving Equation (1) for r_E. Alternatively, if r_E is given and the capital structure is not, one can solve for $\%E$ instead. Having determined the

[7] The equation is shown with only debt and common equity. If the capital structure has preferred equity, add the following term ($r_P \times \%P$) to the right-hand side of the equation.

[8] See, for example, Brealey, Myers and Allen (2017), *Principles of Corporate Finance, 12th Edition*, McGraw-Hill Irwin, New York, pp. 448-453.

ATWACC for the sample companies, I can apply that same ATWACC or an ATWACC adjusted for risk differences to the regulated entity, in this case Vectren.[9]

Q18. Why is the ATWACC relevant to these proceedings?

A18. The ATWACC is one of several procedures in my analysis; it is important because it allows a comparison between the sample companies' costs of capital estimates and the cost of capital for Vectren. Two otherwise identical companies with different capital structures will typically have different costs of equity because the risks to equity holders depend on the financial leverage (i.e., the amount of debt in the capital structure of the company). This makes it difficult to compare cost-of-equity estimates among companies that have different capital structures. The effect of varying financial leverage on the risk-return tradeoffs of companies means that simply averaging individual cost-of-equity estimates across a sample generally does not provide meaningful information about an appropriate representative cost of capital for the industry. Thus it is generally incorrect to compute a sample average return on equity when estimating the cost of capital. However, two otherwise identical companies with different capital structures will generally have comparable ATWACC values. The "apples to apples" comparability of ATWACC across companies with different capital structures makes it a consistent measure of the representative cost of capital in an industry.

Q19. How does the ATWACC approach differ from procedures where the cost of equity and the regulatory capital structure are determined separately?

A19. The ATWACC approach avoids inconsistencies that could arrive from estimating the cost of equity for each of the sample firms without explicit consideration of the financial risk inherent in the market-value capital structure underlying those costs. If the sample's average cost of equity is used to estimate the cost of equity for the company in question, inconsistencies are likely to arise, because this method makes

[9] I refer to the ATWACC to distinguish it from the WACC used in regulatory proceedings which is the weighted-average of the after-tax cost of equity and the *pre-tax* cost of debt instead of the after-tax cost of debt.

no adjustment for any differences among the capital structures of the sample firms used to estimate the cost of equity and the regulatory capital structure used to set rates. Consequently, the sample's estimated return on equity does not necessarily correspond to the financial risk faced by investors in the subject company, in this case Vectren. If the sample's estimated cost of equity were adopted without consideration of differences in financial risk, it could lead to an unjust and inappropriate rate of return.

Q20. Why is it necessary to consider the sample companies' capital structures as well as the regulatory capital structure in your analysis?

A20. Briefly, the cost of equity and the capital structure are inextricably entwined in that the use of debt increases the financial risk of the company and therefore increases the cost of equity. The more debt, the higher is the cost of equity for a given level of business risk. Rate regulation has in the past often focused on the individual components of the cost of capital. In particular, it has treated as separate questions what the "right" cost of equity capital and "right" capital structure should be. The cost of capital depends primarily on the business the firm is in, while the costs of the debt and equity components depend not only on the business risk, but also on the distribution of revenue between debt and equity. The cost of capital is thus the more basic concept. Although the overall cost of capital is constant (ignoring taxes and costs of excessive debt), the distribution of the costs among debt and equity is not. Reporting the average cost of equity estimates from the sample without consideration of the differences in financial risk may result in material errors in the allowed return for Vectren.

Q21. What is the basis for the development of the ATWACC method?

A21. Computing the ATWACC—called the weighted-average cost of capital in textbooks—is the fundamental method used by financial economists to measure the cost of capital. It is a standard topic taught in graduate level courses in corporate finance and is based upon the work of Professors Franco Modigliani and Merton

Miller. Each separately won the Nobel Prize in Economics, in part, for developing the theories underlying the method.

It is critical to keep in mind that the ATWACC method is one useful tool to assist in the analysis of the cost of capital. All cost of capital witnesses estimate the cost of equity using the DCF or the risk positioning models, and all must interpret the results relative to the risk of the regulated company at issue. The purpose of the ATWACC method is to allow an "apples to apples" comparison of the results of the sample companies by adjusting for differences in financial risk due to differences in capital structure. The ATWACC is sometimes mischaracterized in regulatory proceedings and incorrectly criticized, possibly because the critics do not like the method's results, but it is the standard methodology in finance. It is consistent with the use of rate base measured on the basis of original cost (i.e., book value), and does not require a regulator to "rubber stamp" the current market value of the regulated company's stock as is sometimes asserted.

Q22. Is the use of the ATWACC method unconventional?

A22. No. The ATWACC is presented in every textbook on corporate finance of which I am aware.[10] These textbooks calculate the ATWACC in exactly the same way as I do.

Q23. Is the ATWACC approach used by other regulators?

A23. Yes, a number of regulators in the U.S. and in countries around the world rely upon the ATWACC to set rates. Some aspects of the regulatory procedures in these countries may vary, but they all rely upon a book value measure of rate base and a market determined cost of capital to set rates. The countries include the United Kingdom, Australia, New Zealand, and Ireland among others. These countries

[10] See, for example, Brealey, Myers and Allen (2017), *Principles of Corporate Finance, 12th Edition*, McGraw-Hill Irwin, New York, Chapter 19, Ross, Westerfield, Jaffe, and Roberts (2008), *Corporate Finance*, 5th Canadian edition, McGraw-Hill Ryerson, Toronto, Chapter 13, Bodie, Kane and Marcus (2009), *Investments*, McGraw-Hill Irwin, New York, 8th ed., 2009, Chapter 18, and Koller, Goedhart and Wessels (2005), *Valuation*, 4th ed., John Wiley & Sons, Inc. Chapter 5. See Attachment A, Schedule No. D5.17 at 75-91 for the excerpt from *Valuation* textbook.

apparently regard the ATWACC as proper regulatory policy and appropriate for setting rates in a regulatory proceeding.

Q24. What regulators in the U.S. use the ATWACC approach?

A24. Although use of the ATWACC is not prevalent in the U.S., it is used by some regulators. The Surface Transportation Board ("STB") uses the ATWACC method to determine revenue adequacy for railroads, as does the Federal Communication Commission to set rates for local exchange carriers. Florida uses a very similar method to regulate small water companies, and the Colorado Division of Property Taxation uses the ATWACC to value property. The FERC used the ATWACC (calculated as I do) as a discount rate in a valuation dispute.[11] In a decision, the Alabama Public Service Commission said

> [t]he Commission recognizes that the ATWACC analysis is not a prevalent methodology in the United States; however, the focus of that methodology on the relationship between the market value and the associated financial risk of the utility is compelling.[12]

Q25. Is financial risk properly measured by the market value or book value capital structure?

A25. The notion that financial leverage is and should be measured on a market value basis is supported in every textbook on corporate finance of which I am aware.[13] Further, the view is not just an ivory-tower creation. Professional valuation books and guides advocate the use of market value capital structure.[14] Morningstar and Duff and

---

[11] Order Conditionally Accepting Tariff Revisions, Subject to Compliance Filings, Docket No. ER14-2940-000, PJM Interconnection, L.L.C., issued November 28, 2014.

[12] Report and Order, *In re: Public Proceedings established to consider any necessary modifications to the Rate Stabilization and Equalization mechanism applicable to the electric service of Alabama Power Company*, Dockets 18117 and 18416, August 21, 2013, p. 20.

[13] See, *e.g.*, Richard A. Brealey, Stewart C. Myers, and Franklin Allen, 2017, *Principles of Corporate Finance*, 12th edition, McGraw-Hill Irwin, at p. 467; Stephen A. Ross, Randolph W. Westerfield, and Jeffrey Jaffe, 2002, *Corporate Finance*, 6th edition, McGraw-Hill Irwin, at p.386; and Mark Grinblatt and Sheridan Titman, 1998, *Financial Markets and Corporate Strategy,* 1st edition, Irwin/McGraw-Hill, at p. 464.

[14] See, *e.g.*, Tom Copeland, Tim Koller, and Jack Murrin, 2000, *Valuation: Measuring and managing the value of companies*, 3rd edition John Wiley & Sons, p. 204; and Shannon P. Pratt and Alina V.

Phelps—both off-the-shelf cost of capital providers using *Ibbotson* data and analysis—also use market-value capital structure in cost of capital estimates.[15] Similar views were also endorsed by legal decisions on bankruptcy proceedings.[16] Financial risk is a function of the market value capital structure. There is simply no debate in academic or business circles about this point.

Every day experience also indicates that market value is the measure of financial risk. The variability of your return on your investment in your home depends upon the size of your mortgage relative to the appraised (i.e., market) value of your house. For example, if you have a $100,000 mortgage on a house that is worth $200,000 in the current market, you have 50 percent equity in your home. This is true even if the "book value" of the house—the original cost of construction—is only $150,000. It is also the case that the larger the percentage of the appraised value that is financed with a mortgage, the larger will be variability in your equity return as the home value varies. It is the variability of the market value of the house that affects the home owner's risk; the "book value" of the house does not change.

Q26. Can you provide academic evidence that financial leverage is and should be measured on a market value basis?

A26. Yes. The impact of financial leverage on cost of equity has been developed since the 1958 paper by Prof. Franco Modigliani and Merton Miller ("MM"), two economists who eventually won Nobel Prizes in part for their body of work on the effects of debt on firm value.[17] One key corollary of the MM theorems and their various extensions is that cost of equity increases as financial leverage increases. Although the exact

---

Niculita, 2008, *Valuation a business: The analysis and appraisal of closely held companies*, 5th edition, McGraw-Hill, at pp. 216 – 217.

15 See, *e.g.*, Morningstar, *Duff & Phelps 2016 Valuation Handbook – Guide to Cost of Capital*, at p. 15.

16 See, *e.g.*, Bernstein, Stan, Susan H. Seabury, and Jack F. Williams, 2008, "Squaring bankruptcy valuation practice with *Daubert* Demands," *ABI Law Review*, at p. 190.

17 Franco Modigliani and Merton H. Miller (1958), "The cost of capital, corporation finance and the theory of investment," *American Economic Review*, 48, pp. 261-297. See Attachment A, Schedule No. D5.17 at 92-129. For a modern textbook exposition of the capital structure theories, see Brealey, Myers, and Allen, *op cit.*, Chapter 17.

speed of increase in cost of equity differs by models of capital structure, it is universally accepted that as a firm adds debt, its cost of equity increases as a result.

While acknowledging that the cost of equity increases with financial leverage, some people assert that financial risk is measured on a book value basis. This belief is wrong for two reasons. First, in MM's classic paper and subsequent extensions of their original paper, financial leverage has been consistently measured on a market value basis. This is because MM's basic insight is that, under perfect market conditions, financial leverage does not increase the *market value* of a firm as long as different combinations of debt and equity can be selected by the investors themselves.[18] To implement such a self-help financial engineering, investors have to be able to buy and sell debt and equity to achieve their desired combination. The prices at which they transact are, by definition, *market prices.* Second, as a more practical matter, economists generally prefer to use market values because they convey timely information, rather than historical data, about the assets. Business decisions on investment, capital budgeting, and financing are all based on real time market value information.

Q27. Are there any other academic articles that discuss how a company's cost of equity changes as its capital structure changes?

A27. Yes, there are many others. An important example is from Professor Robert S. Hamada, who addressed this issue in "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks."[19] Professor Hamada's adjustment method is consistent with the ATWACC approach, and I present results using this method to provide further insight on the range of ROE estimates after adjusting for financial leverage. I find that the resulting ROE estimates using the Hamada adjustment procedure are similar to those estimates using the ATWACC approach, so the

[18] In developing the theory, MM assume that investors can adjust the capital structures of their portfolios at no cost.

[19] *The Journal of Finance*, Vol. 27, No. 2, Papers and Proceedings of the Thirtieth Annual Meeting of the American Finance Association, New Orleans, Louisiana, December 27- 29, 1971 (May, 1972), pp. 435-452. See Attachment A, Schedule No. D5.17 at 56-74.

Commission should rely on estimates from either procedure to appropriately recognize the impact that differences in leverage have on the cost of equity. Both approaches are widely accepted in academic literature and commonly used amongst finance practitioners. I have included a subset of the academic literature which discusses these financial risk adjustment procedures in Exhibit D5.17.

The alternative Hamada adjustment procedures account for the impact of financial risk recognizing that, under general conditions, the value of a firm can be decomposed into its value with and without a tax shield (Value of Firm = Present Value of Cash Flows without Tax Shield plus Value of Tax Shield).

Assuming that the CAPM is valid, Professor Hamada showed the following relationship between the beta for a firm with no leverage (e.g., 100 percent equity financing) and a firm with leverage is as follows:[20]

$$\beta_L = \beta_U + \frac{D}{E}(1 - \tau_c)(\beta_U - \beta_D) \tag{2}$$

Where β_L is beta associated with the "levered cost of capital"—the required return on assets if the firm's assets are financed with debt and equity—β_U is the beta associated with an unlevered firm—assets are financed with 100% equity and zero debt—, and β_D is the beta on the firm's debt. Finally, τ_c is the corporate income tax rate. Since the beta on an investment grade firm's debt is much lower than the beta of its assets (i.e.,$\beta_D < \beta_U$), this equation embodies the fact that increasing financial leverage (and thereby increasing the debt to equity ratio) increases the systematic risk of levered equity (β_L).

An alternative formulation derived by Harris and Pringle (1985) provides the following equation:

$$\beta_L = \beta_U + \frac{D}{E}(\beta_U - \beta_D) \tag{3}$$

[20] Technically, the relationship requires that there are no additional costs to leverage and that the book value capital structure is fixed.

Unlike Equation (2), Equation (3) does not include an adjustment for the corporate tax deduction. However, both equations account for the fact that increased financial leverage increases the systematic risk of equity that will be measured by its market beta. Both equations allow an analyst to adjust for differences in financial risk by translating back and forth between β_L and β_U. In principle, Equation (2) is more appropriate for use with regulated utilities, which are typically deemed to maintain a fixed book value capital structure. However, I employ both formulations when adjusting my CAPM and ECAPM estimates for financial risk, and consider the results as sensitivities in my analysis.

It is clear that the beta of debt needs to be determined as an input to either Equation (2), or Equation (3). Rather than estimating debt betas, I note that the standard financial textbook of Professors Berk & DeMarzo report a debt beta of 0.05 for A rated debt and a beta of 0.10 for BBB rated debt[21] while other academic literature has reported debt betas of 0.25.[22] I consider this range of 0.05 to 0.25 to be reasonable for debt betas.

Using the estimated debt betas, the levered equity beta of each sample company can be computed (in this case by *Value Line*) from market data and then translated to an unlevered beta at the company's market value capital structure. The unlevered betas for the sample companies are comparable on an "apples to apples" basis, since they reflect the systematic risk inherent in the assets of the sample companies, independent of their financing. The unlevered betas are averaged to produce an estimate of the industry's unlevered beta. To estimate the cost of equity for the regulated target company, this estimate of unlevered beta can be "re-levered" to the regulated company's capital structure, and the CAPM can be reapplied with this levered beta, which reflects both the business and financial risk of the target company.

[21] Berk, J. & DeMarzo, P., *Corporate Finance, 2nd Edition.* 2011 Prentice Hall, p. 389.

[22] "Explaining the Rate Spread on Corporate Bonds," Edwin J. Elton, Martin J. Gruber, Deepak Agarwal, and Christopher Mann, *The Journal of Finance*, February 2001, pp. 247-277. See Attachment A, Schedule No. D5.17 at 130-160.

Hamada adjustment procedures are ubiquitous among finance practitioners when using the CAPM to estimate discount rates.

III. IMPACT OF CURRENT ECONOMIC CONDITIONS

Q28. What is the topic of this section of your testimony?

A28. This section addresses the effect of the current economic situation on the cost of capital and the adjustments to my standard procedures required to estimate the cost of capital more accurately. I also address the effect of the recently enacted Tax Cuts and Jobs Act of 2017 in increasing the risk faced by regulated utilities.

A. ANOMALOUS CAPITAL MARKETS CONDITIONS PERSIST

Q29. Do you believe that capital markets are "back to normal"?

A29. No. Although the Federal Reserve has decided to raise the target range for the federal funds rate to a range of 1 to 1¼ percent since the beginning of 2017[23] and volatility in the financial markets has lessened, economic conditions are not yet back to normal as measured by their status prior to the 2008-2009 credit crisis. For example, although the spreads between U.S. utility bond yields and government bond yields ("yield spread") has narrowed from their peak at the height of the crisis, yield spreads are still elevated relative to the spread before the crisis. This is especially true for lower-rated bonds, including BBB-rated utility bonds. This is, in part, the result of a deliberate policy by the Fed to lower long-term as well as short-term bond yields in an effort to induce investors to move to riskier assets such as stocks.[24]

Q30. Please describe in more detail how the yield spread between U.S. government and utility bonds has changed since the start of the credit crisis.

A30. Although the yield spread on utility bonds has declined from the height of the 2008-2009 credit crisis, the yield spread still remains elevated in relation to pre-crisis levels in response to world economic events and the efforts of the Fed. The yield spread on

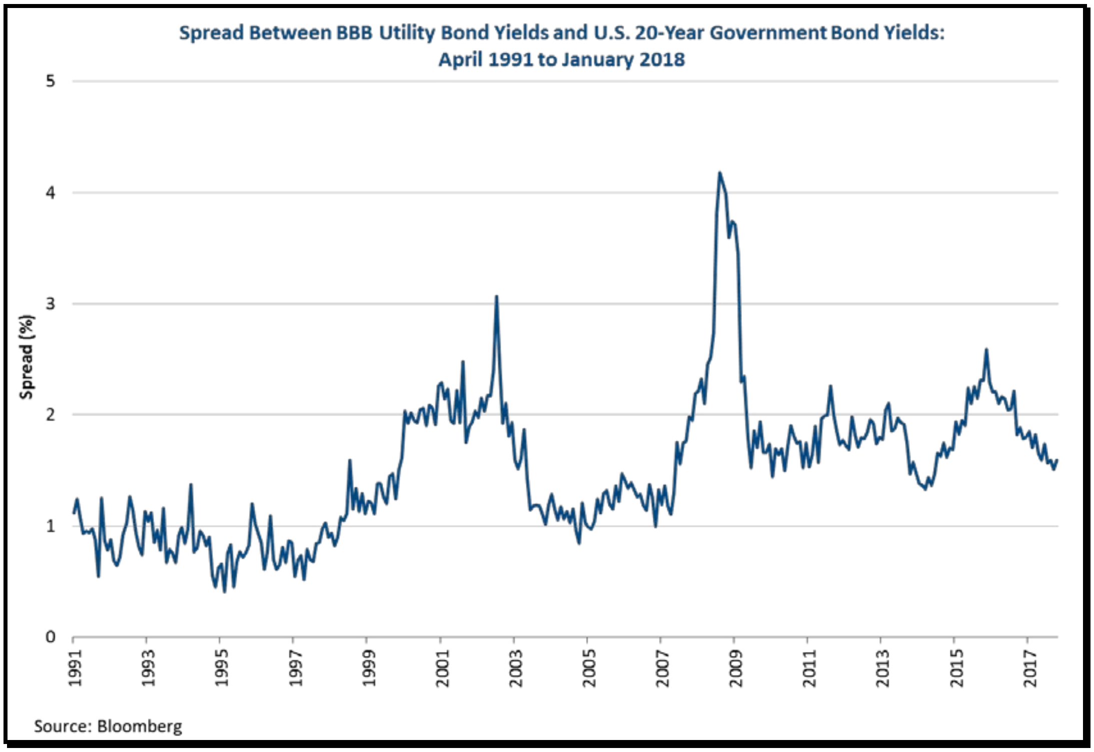
[23] See Federal Open Market Committee, Press Release, September 20, 2017.

[24] *Id.*

utility bonds, such as Bloomberg's BBB-rated utility bonds, has been substantially higher during most of the past eight years than prior to the credit crisis. For example, since the last major peak in November 2008, the spread between the yield on BBB-rated 20-year utility bonds and the yield on 20-year U.S. government bonds, as shown in Figure 2 below, has ranged from a low of 133 basis points to a high of 408 basis points, compared to a historical average of approximately 120 basis points.[25] Additionally, the average yield spread in 2016 of 218 basis points is highly unusual and has reached higher levels in only three of the past 25 years: in 2008 and 2009 during the credit crisis and in 2002 following the collapse of the tech bubble. The yield spread is slightly lower for January 2017 to January 2018 at 170 bps.

[25] Historical average ranges from the beginning availability of U.S. utility bond yield data (April of 1991) through the beginning of the financial crisis (December of 2007) accessed from Bloomberg as of January 31, 2018.

Figure 2
Bond Yield Spreads



In addition to the spike in the spread between utility and government bond yields, the variability in bond yields is also high. BBB utility 20-year bond yields have varied from a high of 4.63 percent to a low of 4.11 percent for a high-to-low difference of approximately 52 basis points over the period January 2017 through January 2018. Table 1 below presents the yield spreads for 20-year utility bonds over several historical periods. Yield spreads have remained elevated compared to historical averages.

Table 1
Comparison of Historical Bond Yield Spreads

Spreads between U.S. Utility Bond (20 year maturity) and U.S. Government Bond (20 year maturity) - %			
Periods	A-Rated Utility and Treasury	BBB-Rated Utility and Treasury	Notes
Period 1 - Average Apr-1991 - 2007	0.93	1.23	[1]
Period 2 - Average Aug-2008 - Jan-2018	1.51	1.98	[2]
Period 3 - Average Jan-2018	1.20	1.59	[3]
Period 4 - Average 15-Day (Jan 10, 2018 to Jan 31, 2018)	1.12	1.51	[4]
Spread Increase between Period 2 and Period 1	0.58	0.75	[5] = [2] - [1]
Spread Increase between Period 3 and Period 1	0.27	0.36	[6] = [3] - [1]
Spread Increase between Period 4 and Period 1	**0.19**	**0.28**	[7] = [4] - [1]

Sources and Notes:
Spreads for the periods are calculated from Bloomberg's yield data.
Average monthly yields for the indices were retrieved from Bloomberg as of January 31, 2018.

Q31. What is the implication of higher than normal yield spreads?

A31. A higher than normal yield spread is one indication of the higher cost of capital prevailing in the capital markets. Investors consider a risk-return tradeoff like the one displayed in Figure 1 (page 8) above and select investments based upon the desired level of risk. The expected return on debt (i.e., the cost of debt) is higher relative to government bond yields than is normally the case even for regulated utilities. Because debt is less risky than equity, the cost of equity is also higher relative to government bond yields than is usually observed. If this fact is not recognized, the traditional cost of capital estimation models will underestimate the cost of capital prevailing in the capital markets.

Q32. Haven't the U.S. stock markets reached record highs and interest rates begun to rise recently?

A32. Yes, the U.S. stock market has been trading at Price-to-Earnings ("P/E") levels which are above historical medians and government bond yields have increased since the U.S. presidential election and the Fed's increase of the federal funds rate. This does not mean, however, that economic conditions are fully back to normal. The recent volatility in the capital market demonstrates that substantial uncertainty remains.

Q33. What further evidence can you provide that U.S. medium- and long-term government bond yields are currently depressed?

A33. Annual yields on long-term U.S. government bonds have continued to be lower than historical values. For instance, the historical average of annual yields on long-term government bonds was 5.23 percent from 1926 to 2010, but the long-term government bond yield declined to just 2.72 percent in 2016.[26] The most recent 15-day average of long-term government bond yield is at 2.77 percent.

Although the U.S. Federal Reserve has discontinued its large-scale asset purchases program, which pushed down yields on medium and long-term U.S. government bonds, it still holds almost $4.4 trillion in assets from this purchasing program.[27] Until there is an intended unwinding of these holdings, uncertainty will persist.

Furthermore, elevated levels of uncertainty in the global capital markets continue to affect the U.S. economy, which remains sensitive to those disruptions. In other words, major capital markets globally have not yet returned to their pre-credit crisis status, and they continue to affect the U.S. capital markets. The European Central Bank (ECB) continues its accommodative stance, which targets a *negative* 0.4% interest rate[28] and continues to purchase billions of euros worth of assets each month (30 billion euros of assets purchased in January 2018),[29] and the Bank of Japan's policy, which has maintained a policy to keep yields on government debt "around zero percent" since September 2016,[30] represent divergent approaches from that

---

[26] See Duff & Phelps's Ibbotson *Stocks, Bonds, Bills, and Inflation ("SBBI") 2017 Valuation Yearbook* at 2-9.

[27] Board of Governors of the Federal Reserve System, Credit and Liquidity Programs and the Balance Sheet, as of February 8, 2018.

[28] European Central Bank, Key ECB Interest Rates, EUROPEAN CENTRAL BANK, https://www.ecb.europa.eu/stats/monetary/rates/html/index.en.html (last visited on February 12, 2018).

[29] European Central Bank, Asset purchase programmes, EUROPEAN CENTRAL BANK, https://www.ecb.europa.eu/mopo/implement/omt/html/index.en.html (last visited February 12, 2018).

[30] See Roger Blitz, Leo Lewis, and Robin Harding, *Nervous investors put the Bank of Japan in the spotlight*, Financial Times, January 16, 2018.

https://www.ft.com/content/f2ec1362-f7ab-11e7-88f7-5465a6ce1a00 .

currently of the Federal Reserve ("Fed"), which halted its asset purchases and has recently decided on a modest increase in interest rates. Dr. Janet Yellen's term as the chairman of the Fed came to a close in early February 2018, and Mr. Jerome Powell has replaced her as chairman. Mr. Powell is expected to maintain Dr. Yellen's policy of gradual interest rate increases. However, uncertainty persists concerning how monetary policy may change with the transition.[31] Finally, increased testing of ballistic missiles by North Korea has had noticeable impacts on the market, such as pushing down yields on 10-year U.S. Treasury Bonds as "investors sought safety."[32]

While U.S. capital markets may currently be benefiting from investors fleeing economic turmoil elsewhere, these global weaknesses underscore investors' lack of confidence in the global economy. These global weaknesses can affect the relatively more stable U.S. economy, and any aggressive action by the Fed on interest rates can easily exacerbate these weakened global economies, which in turn may affect U.S. capital markets.

Q34. Are interest rates and treasury yields expected to rise in the future?

A34. Yes. Since the beginning of 2017, the Fed has increased the federal funds target interest rate three times, which has increased yields on U.S. Treasury notes briefly, but for many reasons discussed above, yields on 30-year U.S. Treasury bonds are currently lower than at the beginning of 2017. While yields on the 10-year Treasury bond have increased from 2.43 percent in January 2017 to 2.8 percent in early February 2018, yields on the 30-year Treasury bond have declined from 3.02 percent to 2.88 percent.[33] However, economists and investors do not expect yields to persist at these unprecedented low levels indefinitely. According to the *Blue Chip Economic*

---

[31] See Heather Long, *Who is Jerome Powell, Trump's pick for the nation's most powerful economic position?*, Washington Post, November 2, 2017.

https://www.washingtonpost.com/news/wonk/wp/2017/10/31/jerome-powell-trumps-pick-to-lead-fed-would-be-the-richest-chair-since-the-1940s/?utm_term=.d9e7ae80ab87.

[32] See *Financial Times* article "Flight to havens after North Korea missile launch", https://www.ft.com/content/5dab7a38-8c56-11e7-a352-e46f43c5825d.

[33] Bloomberg accessed as of January 31, 2018.

Indicators report dated October 10, 2017, the consensus economic projections for the yield on 10-year U.S. Treasury notes are 3.5 percent on average in 2019 to 2023 and 3.7 percent on average from 2024 to 2028.[34] These forecasts are substantially higher than the current yield on 10-year U.S. government notes.[35] This highlights the fact that current long-term and medium-term U.S. government bond yields are low relative to historical levels as well as compared to consensus forecasts of future rates. The unusually low current long-term government bond yields, along with elevated yield spreads due to risk aversion, must be considered when evaluating the results of the risk-positioning model, because the downward bias in the long-term risk-free interest rate will inappropriately lower the sample companies' ROE estimates generated by the CAPM method.

Q35. How do you adjust your cost of capital estimation methods to correct for current economic conditions?

A35. I make no adjustment to the DCF method. For the risk positioning method, I recognize the larger than average yield spreads on utility debt by adding a "yield spread adjustment" to the current long-term risk-free rate. This has the effect of increasing the intercept of the Security Market Line displayed in Figure 1 (page 8) above. I also present results from the risk positioning model by increasing the MRP over the 6.94 percent historical MRP. This has the effect of increasing the slope of the Security Market Line displayed in Figure 1 (page 8) above. I present a sensitivity test of the effect of an increase in the MRP to 7.94 percent, and yield spread adjustments of 20 basis points ("bps"). Table 4 (page 52) below lists the parameters of these two scenarios.

---

[34] See *Blue Chip Economic Indicators*, dated October 10, 2017, page 14.

[35] See Schedule D5.9.

Q36. How do you estimate the increase in MRP needed to adjust for the increased cost of capital stemming from the current market turmoil?

A36. Estimating the MRP is always imprecise and controversial. Measuring the change in MRP due to the current economic situation is likely to be no different, but it is still necessary to estimate the MRP as carefully as possible given the change in economic conditions. Fortunately, there is a way to provide a quantitative benchmark for the required increase in MRP based upon a paper by Edwin J. Elton, et al., which documents that the yield spread on corporate bonds is normally a combination of a default premium, a tax premium, and a systematic risk premium.[36] As displayed in Table 1 (page 22) above, the yield spreads for A-rated and BBB-rated utility debt are currently elevated compared to the average for the period 1991-2007.

Q37. How do you use the information in Table 1 (page 22) concerning the increase in yield spreads to estimate the increase in the MRP?

A37. Table 1 (page 22) shows that recent yield spreads for A-rated and BBB-rated utility debt have increased by about 20 bps and 30 bps respectively for 20-year maturities. This means that investors require a higher return on investment grade utility debt relative to the return on U.S. Government debt than before the credit crisis. Some of the increase in yield spread for A-rated debt may be due to an increase in default risk (although this is more likely a component of the larger increase in BBB-rated utility spreads).[37] The increase in A-rated utility yield spread is due to a combination of an increase in the systematic risk premium on A-rated debt and the downward pressure on the yield of risk-free debt due to the flight to safety. The increase in the default risk premium for A-rated debt is undoubtedly very small because A-rated utility debt has not been at the center of the wave of defaults based upon collateralized mortgage debt. This means that the vast majority of the increase in yield spreads is due to a

[36] "Explaining the Rate Spread on Corporate Bonds," Edwin J. Elton, Martin J. Gruber, Deepak Agarwal, and Christopher Mann, *The Journal of Finance*, February 2001, pp. 247-277. See Attachment A, Schedule No. D5.17 at 130-160.

[37] Although there is no increase in tax premium due to coupon payments, there may be some increase due to a small tax effect resulting from the probability of increased capital gains taxes when the debt matures.

combination of the increased systematic risk premium and the downward pressure on the yields of government debt. In other words, either the MRP has increased or the risk-free rate is under estimated, or, alternatively, both. In my analysis, I assume that there has been at least a 20 bps increase in utility spreads, due to either an increase in the MRP (which drives the increase in systematic risk premium), or to downward pressure on the risk-free rate. While this is slightly higher than the observed 19 bps increase in the yield spread over the latest 15 days, I believe this estimate is conservative when the recent downturn in the stock market is considered.

Q38. How do you allocate the increase in the yield spread (not due to the estimated increase in default risk) to the increase in systematic risk or to the under estimation of the risk-free rate due to downward pressure on government bond yields?

A38. There is no precise way to allocate the increase in yield spread between the increase in systematic risk and the underestimation of the risk-free rate arising from downward pressure on government bond yields; however, assuming a debt beta of 0.25[38] means that an increase in the MRP of one percentage point translates into a ¼ percentage point increase in the risk premium on debt (i.e. 0.25 (beta) times 1 percentage point (increase in MRP) = ¼ percentage point). The relationship among the increased yield spread for A-rated utilities ($\Delta\, spread$), the underestimation of the expected risk-free rate (Δ), and the required adjustment to the market risk premium ($\Delta\, MRP$) can be represented as follows.

$$\Delta spread - \Delta = 0.25 \cdot \Delta MRP$$

A 25 bps increase in the yield spread is therefore consistent with a 100 bps increase in the MRP if there were no underestimation of the risk free rate. Alternatively, it could represent an underestimation of the risk-free rate. The greater the increase in yield spread attributed to an increase in systematic risk, the larger the corresponding

[38] Elton, *et al*. estimate the average beta on BBB-rated corporate debt as 0.26 over the period of their study, and A-rated debt will have a lower beta than BBB-rated debt.

increase in the MRP and the smaller the effect of the downward pressure on the risk-free rate.

I consider two scenarios in my analysis. In the first scenario, I attribute the 20 bps increase in the yield spread entirely to an underestimation of the risk-free rate. In other words, a 20 bps increase in the yield spread is consistent with a 20 bps underestimation of the risk-free rate, assuming that none of the change in yield spread is driven by an increase in systematic risk. In the second scenario, I attribute a slightly higher 25 bps increase in the yield spread entirely to an underestimation of the MRP.[39]

Q39. Would the estimate of the effect of an increase in the MRP be different if the estimate of the beta of an A-rated bond were different?

A39. Yes. If the beta of an A-rated bond were higher, the increase in the systematic risk premium in the yield spread for each one percentage point increase in the MRP would be smaller. Alternatively, if the beta of an A-rated bond were lower, the increase in the systematic risk premium in the yield spread for each on percentage point increase in the MRP would be larger.[40] However, I believe that a beta estimate of 0.25 for A-rated utility debt is reasonable for this purpose, because the debt of any company is less risky than its equity. A beta estimate of 0.25 for A-rated utility debt is likely to be conservative, especially when compared to an average estimated beta of 0.75 (*Value Line* average beta) for the expanded sample. Moreover, a beta estimate of 0.25 is no doubt conservative because if the estimated beta were lower (as is likely) then the increase in the MRP necessary to result in a 20 bps increase in the yield spread would be higher. As noted above, the average estimated beta for BBB-rated debt was 0.26 at the time of the Elton et al study, and A-rated debt will have a lower estimated beta. Even if the average beta for BBB-rated debt is higher today than at

[39] The increase in the yield spread for BBB-rated utility debt is 28 bps and the beta of debt could easily be less than 0.25 so a 100 bps increase in the MRP is reasonable..

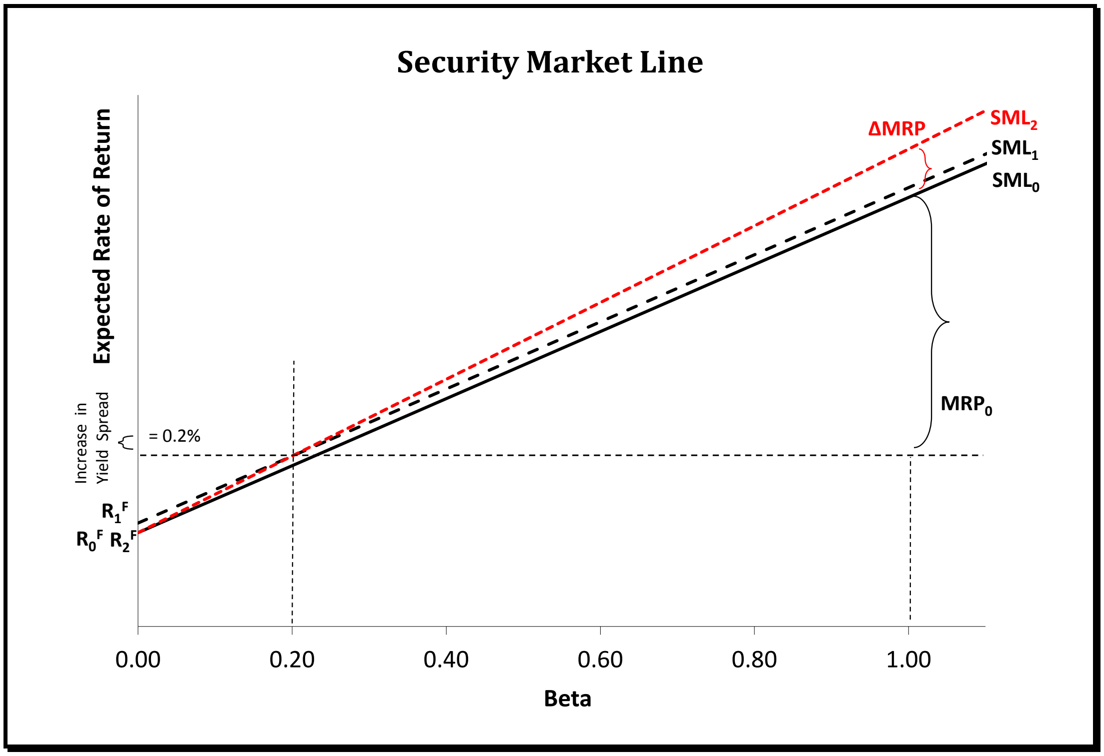
[40] As noted above, the Berk and DeMarzo textbook reports average debt betas for A-rated debt to be 0.05.

the time of the Elton et al study, it is likely that an estimate of 0.25 for A-rated debt is reasonable.

Q40. Would you provide a graph of how the scenarios you consider affect the Security Market Line?

A40. Yes. See Figure 3 below. Scenario 1 (shown as SML_1 in Figure 3) attributes the entire increase in the yield spread on A-rated utility debt to underestimation of the risk free rate by shifting the Security Market line up in parallel fashion by 20 bps ($R_1^F - R_0^F$). Scenario 2 (shown as SML_2 in Figure 3) attributes the increase in the yield spread to an increase in the MRP by increasing the slope of the line by 1.0 percentage points (ΔMRP).

Figure 3
Security Market Line under Two Scenarios



Q41. Can you summarize your thoughts with regard to the MRP and the financial crisis?

A41. Yes. There remain serious concerns of a very slow growth recovery. Economic and political uncertainty continues in countries around the world, in an increasingly global economy. It is difficult to believe that the MRP has not increased from its level in more normal times, whether there is any particular agreed model for how to calculate the increase or not.

In light of these circumstances and the calculations described above, I submit that a 100 bps increase in the MRP presents a reasonable span of the adjustments that might be made. As discussed in the Empirical CAPM estimation below, I have analyzed two scenarios with alternative adjustments to the risk-free rate and the MRP. These scenarios recognize the simple reality that while the financial turmoil and interventions by the Fed and the U.S. government have made it more difficult to measure the cost of equity accurately, the required return on equity has increased, not decreased, as a naïve, mechanical implementation of the models might suggest.

Q42. What is the current evidence regarding market volatility?

A42. A measure of the market's expectations for volatility is the VIX, which measures the 30-day implied volatility of the S&P 500 index. This index is sometimes called the "investor fear gauge"[41] because it provides a market indication of how investors in stock index options perceive the likelihood of large swings in the stock market within the next month. As of February 7, 2018, the VIX stood at 28, substantially higher than the 1990-present average of 19 or the two year average of 13.5.[42]

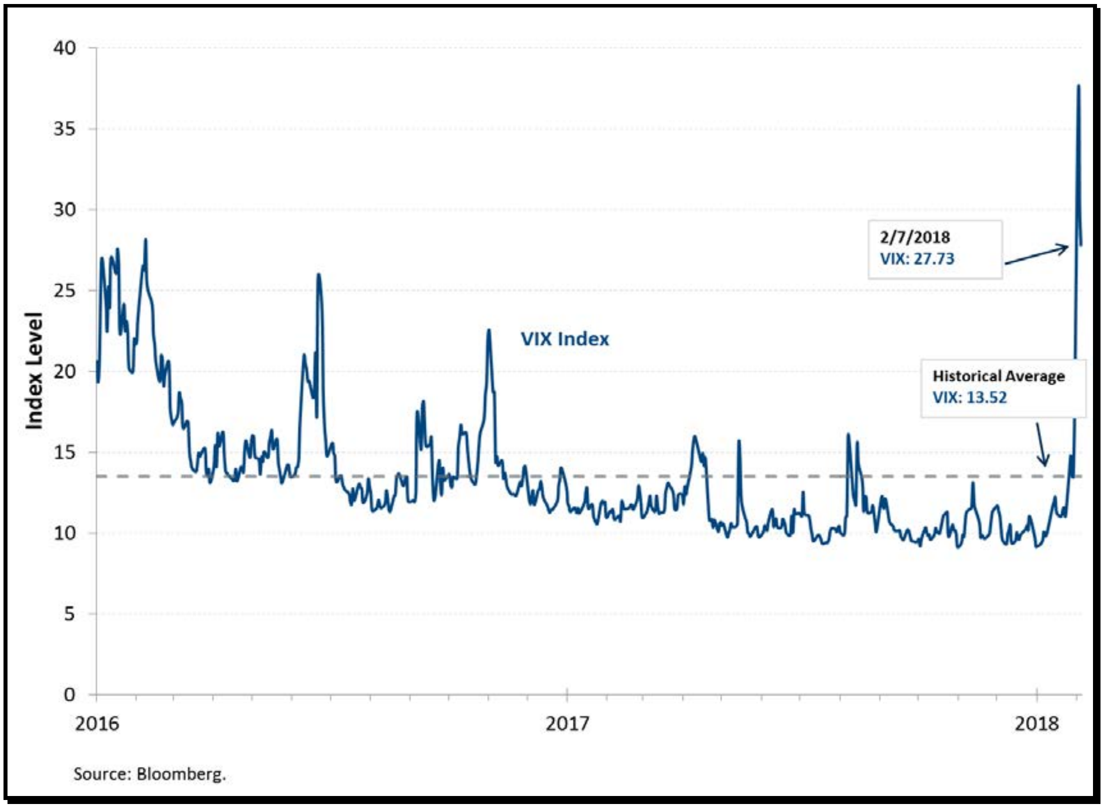
In 2016 and 2017, the VIX displayed considerable short-term volatility. During that period the index reached as high as 28 and fell as low as 9. At the end of January 2018, the VIX stood at 13.5. However, it increased dramatically during the first week of February, reaching as high as 37. This demonstrates that, consistent with recent movements in the stock market, investors expect a high level of market volatility over the coming 30 days.

---

[41] See Rachel Koning Beals, *Stock market 'fear gauge' VIX remains up over 20% in wake of latest North Korean action*, MarketWatch, August 29, 2017.

[42] Bloomberg as of February 7, 2018.

Figure 4
Historical VIX Levels

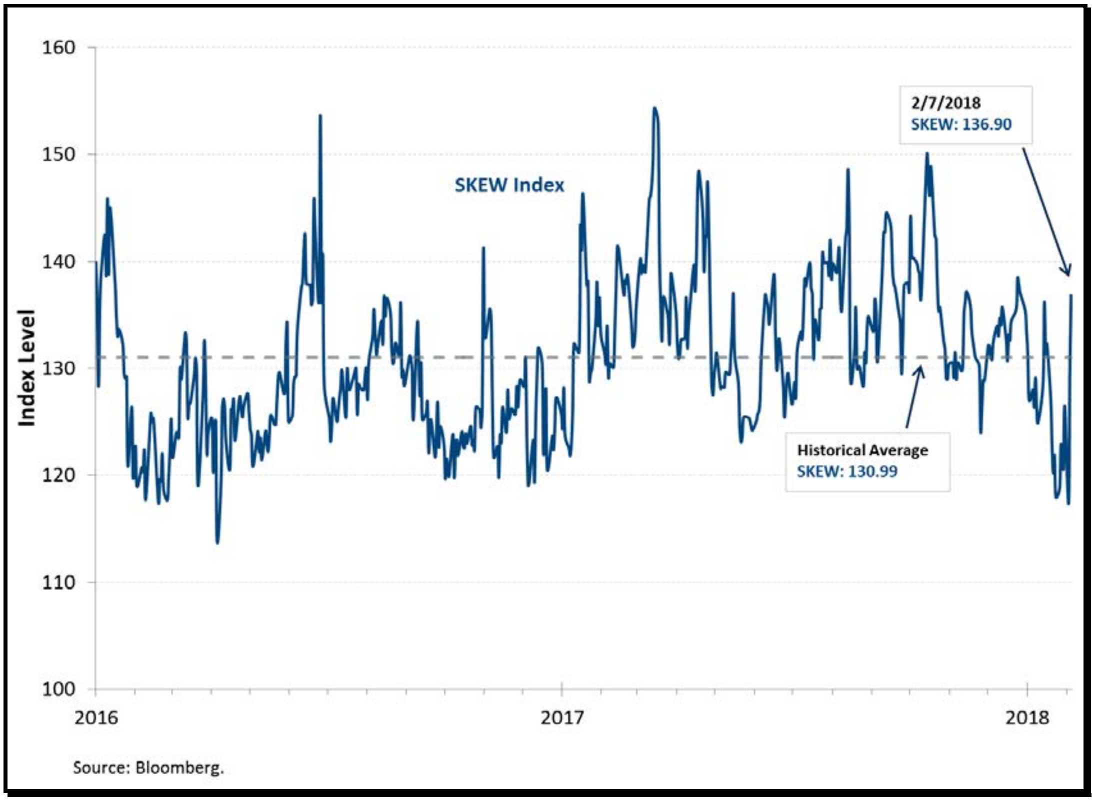


Q43. Are there other indications that investors are exhibiting elevated signs of risk aversion?

A43. Yes, the SKEW index measures the market's willingness to pay for protection against negative "black swan" stock market events (i.e., sudden substantial downturns). A SKEW value of 100 indicates outlier returns are unlikely, but as the SKEW value increases, the probability of outlier declines also increases. The SKEW currently stands at almost 137, while the index has averaged 119 since 1990, and 131 in the past two years.[43] This indicates that in addition to short-term volatility expectations being low, investors are exhibiting signs of elevated risk aversion over concerns of downside tail risk.

43 Bloomberg as of February 7, 2018.

Figure 5
Historical SKEW Levels



B. THE NEW TAX LAW INCREASES RISKS FACING REGULATED UTILITIES

Q44. How will the Tax Cuts and Jobs Act of 2017 affect regulated utilities?

A44. The Tax Cuts and Jobs Act of 2017 (Public Law 115-97) ("TCJA"), signed into law on December 22, 2017, reduces the federal corporate marginal tax rate from 35 percent to 21 percent. Although the tax law is likely to be a net positive for investors in unregulated companies, it is likely that customers, rather than shareholders, of regulated companies will reap the majority of the benefits because the savings in income taxes will flow through to customers. The reduction in income tax will likely increase the risks facing regulated companies because the effect of the law will be a reduction in their cash flows.

Q45. How will the TCJA reduce the cash flows of regulated companies?

A45. The law can reduce cash flows for regulated companies in several ways. First, the reduction in the corporate tax rate reduces the income tax allowance needed, i.e., the ROE "gross up" for income tax is smaller. This results in a reduced revenue requirement and decreased pre-tax cash flows. Second, on an after tax basis, the benefit of any accelerated tax depreciation will go down in proportion to the reduction in tax rate, leading to a reduction in after-tax cash flows. Third, regulated utilities will need to refund Excess Deferred Income Taxes ("EDIT") to their customers through lower rates. The creation of EDIT relates to Accumulated Deferred Income Tax ("ADIT"), which represents the timing difference in depreciation for income tax and regulatory purposes. Typically, depreciation for tax purposes is accelerated relative to regulatory depreciation so that Deferred Income Tax "DIT" is positive in the early years of a regulated asset's life and negative in the later years. The assumption is that ADIT will be zero for any asset at the end of its regulatory life; however, that would not be true with a change in the corporate tax rate, unless EDIT is addressed. Because of the reduction in the corporate tax rate, the excess ADIT becomes EDIT that will be refunded to customers over the remaining life of the asset. As the EDIT is amortized, it will increase the rate base, but on net the return of EDIT will reduce the utility's cash flows, both before and after taxes, until the EDIT has been exhausted.[44] Finally, the law eliminates bonus depreciation. Bonus depreciation allows utilities to recognize additional depreciation for tax purposes during the first year of an asset's operation. While bonus depreciation reduces rate base, it creates an upfront increase in a utility's cash flows in the form of lower tax payments. Thus, the elimination of bonus depreciation will negatively impact some utilities' after tax cash flows.

[44] This is true because the return on a dollar of increased rate base is less than the cash flow from a dollar of depreciation.

Q46. How will the TCJA 2017 affect the expected volatility of cash flows for regulated companies?

A46. This example assumes that the revenue requirement has been adjusted to account for the lower corporate income tax rate. For regulated companies, the change in the income tax allowance will result in greater volatility of net income (and cash flow) because the regulatory income tax allowance provides a "buffer" against the impact of variations in expected costs and expected revenue on net income. Consider for example the effect on net income of a 10 percent increase in sales. All else equal, net income would increase by about 6.5 percent for a 35 percent income tax rate, (i.e. 0.10 times (1 – 0.35)), but would increase by 7.9 percent for a 21 percent income tax rate. The change would be similar for a decrease in revenue. Moreover, the variation in net income is likely to be systematic in that variations in revenue are generally related to variations in the economy. Recall that systematic risk is the type of risk that affects the cost of capital.

Q47. How will the TCJA affect a regulated company's credit metrics?

A47. Credit metrics are likely to be negatively impacted due to a reduction in the regulated utilities' cash flow because cash flow metrics are closely observed by the ratings agencies. The reduction in income tax allowance, the expected refunds of EDIT, and the loss of bonus depreciation will reduce cash flow. Yet the tax reform has not impacted the amount of assets, a portion of which will be debt-financed, necessary to serve the utilities' customers. Decreases to the cash flow metrics, such as cash flow to debt ratios closely monitored by credit rating agencies to inform their credit opinions, negatively impacts the credit profile of many regulated utilities.[45] These effects suggest that the allowed ROE, the amount of equity in the capital structure, or

[45] "Moody's changes outlooks on 25 US regulated utilities primarily impacted by tax reform," Moody's Investor Service, Global Credit Research, January 19, 2018, and "Tax reform is credit negative for sector, but impact varies by company," Moody's Investor Service, Sector Comment, January 24, 2018. Also "U.S. Tax Reform: For Utilities' Credit Quality, Challenges Abound," S&P Global Ratings, Rating Direct, January 24, 2018; and "Tax Reform Impact on the U.S. Utilities, Power & Gas Sector: Tax Reform Creates Near-Term Credit Pressure for Regulated Utilities and Holding Companies," Fitch Ratings, Special Report, January 24, 2018.

possibly both should be increased to offset the negative effects of the income tax law. While the uncertainty surrounding the passage of a tax reform bill has been removed, it is unlikely that these impacts on the cost of capital will immediately appear in the estimation models. The law has not yet been in place for even one fiscal quarter. A longer period of market data and updates of analyst forecasts is needed before the cost of capital estimation models will begin to show the impacts of the new tax law.

IV. SAMPLE SELECTION

A. THE EXPANDED SAMPLE

Q48. What factors do you consider in selecting a proxy group?

A48. The cost of capital for any part of a company depends on the risk of the lines of business in which the part is engaged, not on the overall risk of the parent company on a consolidated basis. According to financial theory, the overall risk of a diversified company equals the market-value weighted average of the risks of its components, so selecting a sample concentrated in the regulated company's line of business is important. Vectren is a regulated gas distribution utility. Currently there is available only a relatively small sample of publicly-traded gas distribution utilities (five companies) whose primary business is distribution of natural gas under cost of service regulation and which meet my standard set of criteria for M&A activity.

Q49. What additional selection criteria did you apply?

A49. The companies must own substantial regulated assets, must not exhibit any signs of financial distress, and must not be involved in any substantial merger and acquisition ("M&A") activities that could bias the estimation process.[46] In general, this requires that over a five year study period and up to the date of the analysis, the sample

[46] This includes pending (but announced) M&A activity but adjusts for M&A activity that does not appear to bias the beta estimates substantively, (such as small, spaced-out transactions, transactions involving multiple parties or parent drop-downs). Notably, I include New Jersey Resources and South Jersey Industries, which were recently engaged in M&A, WGL Holdings, which is currently a target for acquisition by AltaGas, and Spire which engaged in large acquisitions in 2013 and 2014. My reasons for including these companies are explained in greater detail in my testimony.

companies have an investment grade credit rating, a high percentage of regulated assets (greater than 50 percent),[47] no significant merger activity, no dividend cuts, and no other activity that could cause the growth rates or beta estimates to be biased. Finally, I require that data from S&P or Moody's, *Value Line*, and Bloomberg—each widely known and utilized by investors—be available for all sample companies.

Q50. Can you summarize how you selected the expanded sample?

A50. I formed the sample from the universe of publicly traded natural gas distribution utilities as classified by the *Value Line Investment Survey Plus Edition*.[48] This resulted in an initial group of 17 companies. I then eliminated companies by applying additional selection criteria designed to remove companies with unique circumstances which may bias the cost of capital estimates. This ultimately yielded only five natural gas LDCs, which is too few for statistical reliance. Therefore, I expanded the initial sample to include certain gas LDCs involved in M&A activity during the last 5 years. This added 4 more utilities after screening for the criteria described below for a total of 9 companies in the expanded sample.

Q51. Why is it appropriate to expand the gas sample with companies with some M&A activity?

A51. The ideal sample would consist of regulated gas LDCs with no M&A activity during the past 5 years. Because my original screen yielded only 5 companies, I reviewed the data for gas LDCs involved in M&A activity during the last 5 years. This led me to add four additional companies to my sample – Spire, New Jersey Resources, South Jersey Industries, and WGL Holdings. Three years ago, Spire engaged in M&A that

---

[47] I use the Edison Electric Institute's methodology used for classification of electric utilities to determine the percentage of assets classified as regulated, mostly regulated or diversified, for the gas LDC companies in my sample. Specifically, and consistent with Edison Electric Institute's methodology, I applied the following asset percentage thresholds: Regulated - greater than 80 percent of total assets are regulated; Mostly Regulated - 50 to 80 percent of total assets are regulated; Diversified - less than 50 percent of total assets are regulated. I used company asset information as reported by S&P Capital IQ as of August 24th, 2017 or from the companies' most recent 10K for performing my calculation of asset classification for the sample companies.

[48] The 17 companies are from *Value Line Investment Analyzer*, accessed as of November 9, 2017.

doubled the size of the company. While this would not affect the DCF analysis, it could affect the CAPM analysis. Based on a review of Bloomberg 3- and 5-year Betas for Spire, I concluded the merger had not materially affected the company's Beta.[49] Thus, I included it in both my DCF and CAPM estimates.

In April 2017, New Jersey Resources and South Jersey Industries announced interest in a merger. However, the parties subsequently terminated negotiations in October 2017. Moreover, the merger announcement had a small impact on the companies' equity valuations relative to general price movements in the equity market. In January 2017, AltaGas announced a still-pending acquisition of WGL Holdings. However, the announcement had a small impact on the company's equity valuations relative to general price movements in the equity market. For these reasons, I included New Jersey Resources, South Jersey Industries, and WGL Holdings in my full sample. To verify the appropriateness of including these companies, I also considered a subsample that excluded them.

B. COMPARISON OF VECTREN TO THE EXPANDED SAMPLE COMPANIES

Q52. What are the characteristics of the expanded sample companies you have chosen?

A52. The expanded sample is comprised of regulated companies whose primary source of revenues and majority of assets are in the regulated portion of the natural gas distribution industry. The final sample consists of the nine regulated natural gas LDCs listed in Table 2 below.

Q53. Can you describe the financial and regulatory characteristics of the sample in comparison to Vectren?

A53. Table 2 below reports the sample companies' annual revenues for the trailing twelve months ended December 2017 and the percentage of their assets devoted to regulated operations according to EEI's classifications of being either regulated ("R"), having

[49] Using both 3 and 5 years of historical data, Bloomberg reports a Beta of 0.64 for Spire.

greater than 80 percent regulated assets or mostly regulated ("M"), having 50-80 percent regulated assets. Table 2 also displays the Market Capitalization and the S&P Credit Rating for each company as of December 31, 2017, and the weighted average long-term (5-year) earnings growth rate estimate from Thomson Reuters IBES and *Value Line* for all of the companies in the expanded sample.

Table 2
Financial Characteristics of the Expanded Sample

U.S. Gas Sample

Company	DCF Subsample	Annual Revenues (USD million)	Regulated Assets	Market Cap. 2017 Q4 (USD million)	Betas	S&P Credit Rating (2016)	Long Term Growth Est.
	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Atmos Energy	*	$2,868	R	$9,303	0.70	A	5.4%
Chesapeake Utilities	*	$585	M	$1,293	0.70	na	12.2%
ONE Gas Inc.	*	$1,519	R	$3,901	0.70	A	7.0%
South Jersey Inds.		$1,233	M	$2,516	0.85	BBB+	14.8%
Southwest Gas	*	$2,450	R	$3,860	0.80	BBB+	7.8%
Spire Inc.	*	$1,821	R	$3,677	0.70	A-	4.7%
New Jersey Resources		$2,292	M	$3,499	0.80	na	1.9%
Northwest Natural Gas	*	$764	R	$1,776	0.70	A+	8.8%
WGL Holdings Inc.		$2,388	R	$4,392	0.80	A	-0.4%
Full Sample Average		**$1,769**		**$3,802**	**0.75**		**6.9%**
Subsample Average		**$1,668**		**$3,968**	**0.72**		**7.6%**

Sources and Notes:
[1]-[2]: Denotes companies used in the CAPM and DCF subsamples.
[3]: Bloomberg as of January 31, 2018. Most recent four quarters.
[4]: See Table No. MV-GAS-2. Key:
R - Regulated (More than 80% of assets regulated).
M - Mostly Regulated (50%-80% of assets regulated).
[5]: See Table No. MV-GAS-3 Panels A through I.
[6]: See Supporting Schedule # 1 to Table No. MV-GAS-10.
[7]: S&P Credit Ratings from Research Insight as of 2017 Q4. Research Insight does not report S&P credit ratings for MGE Energy. I use the S&P ratings of MGEE's subsidiary, Madison Gas and Electric Company.
[8]: See Table No. MV-GAS-5.

Q54. How does the business risk of Vectren compare to that of the sample?

A54. Vectren's business is concentrated in regulated natural gas distribution services. Its annual revenues are $2.6 billion with a market capitalization of about $5.5 billion, so it is slightly larger than the average company in the sample. Vectren's beta is 0.75 which is the sample average. Regulatory policy plays a role in the business risk of the Company. It also has a credit rating of A- which is comparable to those of the sample companies, but Vectren's credit rating outlook has been revised to negative from stable due to the negative expected effect of the TCJA and due to the

Company's large capital spending plan.[50] Vectren's service is heavily dependent upon manufacturing and heavy industry as well as the ongoing viability of Wright Patterson Air Force Base. Vectren's unique risks are discussed further in the testimony of Company witness, Colleen Ryan.

C. CAPITAL STRUCTURE

Q55. What regulatory capital structure is Vectren requesting in this proceeding?

A55. Vectren had a regulatory capital structure consisting of approximately 50.6 percent equity and 49.4 percent debt as of December 31, 2017,[51] as supported by company witness Patrick Edwards and set forth in Schedule D-1A. The expanded sample averages about 51 percent equity and 49 percent debt on a book basis. The highest percent of book equity for the companies in the sample is 62 percent equity (ONE Gas Inc.) and the lowest is 43 percent equity (WGL Holdings Inc.). My recommended range for ROE is a function of Vectren's capital structure, the sample average ATWACC estimates, the Hamada adjustment procedures, and the relative risk of the Company compared to the sample.

V. COST OF CAPITAL ESTIMATES

Q56. How do you estimate the sample companies' costs of equity?

A56. As noted earlier, I apply two general methodologies—risk positioning and DCF—both of which are standard ways of estimating a company's cost of equity. For my CAPM (risk positioning) based estimates, I consider a range of sensitivities to reflect well-documented empirical deficiencies in the CAPM when used in conjunction with an equity market index. These sensitivities are called the Empirical CAPM. I also report results generated by two versions of the DCF approach: the single-stage and the multistage DCF models.

---

[50] S&P Global Ratings, *RatingsDirect*, "Vectren Corp. and Subsidiaries Outlooks Revised To Negative From Stable; 'A-' Ratings Affirmed," March 9, 2018.

[51] By regulatory capital structure, I mean the capital structure used to set rates in this proceeding.

A. THE CAPM-BASED ESTIMATES

Q57. Can you explain the CAPM?

A57. Modern models of capital market equilibrium express the cost of equity as the sum of a risk-free rate and a market risk premium. The CAPM is the longest-standing and most widely used of these theories. To implement the model requires specification of (1) the current values of the benchmarks that determine the Security Market Line [see Figure 1, (page 8)]; (2) the relative risk of a security or investment; and (3) how the benchmarks combine to produce the Security Market Line. Given these specifications, the company's cost of capital can be calculated based on its relative risk. Specifically, the CAPM states that the cost of capital for an investment, S (e.g., a particular common stock), is given by the following equation:

$$r_s = r_f + \beta_s \times MRP \quad (4)$$

where r_S is the cost of capital for investment S;
r_f is the risk-free interest rate;
β_S is the beta risk measure for the investment S; and
MRP is the market risk premium.

The CAPM relies on the empirical fact that investors price risky securities to offer a higher expected rate of return than safe securities. It says that the Security Market Line starts at the risk-free interest rate (that is the return on a zero-risk security, the y-axis intercept in Figure 1 (page 8), equals the risk-free interest rate). Further, it says that the risk premium of a security over the risk-free rate equals the product of the beta of that security and the risk premium on a value-weighted portfolio of all investments, which by definition has average risk.

1. The Risk-free Interest Rate

Q58. What interest rates do your calculations require?

A58. Modern capital market theories of risk and return (e.g., the theoretical version of the CAPM as originally developed) use the short-term risk-free rate of return as the starting benchmark, but regulatory bodies frequently use a version of the risk positioning model that is based upon the long-term risk-free rate. In this proceeding, I rely upon the long-term version of the risk positioning model. Accordingly, the implementation of my procedures requires use of long-term U.S. Treasury bond interest rates. For this reason, I use a risk-free rate based on the forecasted value from *Blue Chip Economic Indicators*. Specifically, I use the 3.4 percent yield on the 10-year U.S Treasury bond forecasted to be in effect in 2019,[52] and adjust upward by 54 bps, which is my estimate of the representative maturity premium for the 20-year over the 10-year Treasury Bond. The resulting value for the unadjusted risk-free rate is 3.94 percent.

Q59. Why didn't you use the version of the CAPM that relies on the short-term risk-free rate in this proceeding?

A59. Short-term Treasury bill yields remain at artificially low levels due to the efforts of the Fed to stimulate the economy. As a result, the risk positioning required ROE estimates using the short-term Treasury bill yields as the risk-free interest rate are unreasonably low. For example, the estimates are sometimes less than the corresponding company's current market cost of debt, which is unreasonable. A company's equity is always riskier than its debt and requires a higher expected return, because debt holders are paid before equity holders in the event of bankruptcy or other financial distress.

2. The Market Risk Premium

Q60. Why is a risk premium necessary?

A60. Experience (e.g., the recent credit crisis in stock markets worldwide and the U.S. market's October Crash of 1987) demonstrates that shareholders, even well-diversified shareholders, are exposed to enormous risks. By investing in stocks instead of risk-free government Treasury bills, investors subject themselves not only to the risk of earning a return well below that which they expected in any year but

[52] *Blue Chip Economic Indicators*, dated October 10, 2017.

also to the risk that they might lose much of their initial capital. This is fundamentally why investors demand a risk premium.

Q61. Has the estimate of the MRP been controversial over the recent past?

A61. Yes. Historically, the appropriate method to estimate the MRP was to consider the historical average realized return on the market minus the return on a risk-free asset over as long a series of time as possible; however, this procedure came under attack during the period of time generally referred to as the "tech bubble" when the stock markets in the U.S. reached very high valuation levels relative to traditional metrics of value. The period of the tech bubble also resulted in the average realized return on the market increasing to a very high level. Attempts to explain the high stock market valuation levels centered on the hypothesis that the MRP must be dramatically lower than previously believed, but this hypothesis conflicted with the fact that realized returns over the period were very high. The result was an academic debate on the level of the forward-looking MRP and how best to estimate it—a debate that has still not been fully resolved. As discussed in Section III, stock markets declined as a result of the credit crisis, and stock prices became extremely volatile. It is likely the MRP is now higher than the historical average realized return on the market minus the return on the risk-free asset.

Q62. How do these factors affect the cost of capital for the Company?

A62. The Company invests in long-lived assets which cannot be easily liquidated (they are hard physical assets that once put in place cannot easily be moved). Investment is a voluntary activity, and investors generally require an expected return that is consistent with the risk they take on; therefore, it could damage the ability to access capital if investors view the allowed rate of return as lower than the required rate of return. The problem is not avoided for subsidiary companies that are 100 percent parent owned because the parent company must consider the opportunity cost of capital when making investments. Investors expect managers to invest in projects which provide expected returns at least equal to the cost of capital.

Q63. What is your conclusion regarding the MRP?

A63. Historically, much of the controversy over market risk premium centered on various reasons why it may not be as high as frequently estimated. Although none of the arguments were completely persuasive in and of themselves, I generally gave some weight to these issues in past testimony and reduced my estimate of the MRP. Conversely, recent events have strongly suggested an increase in the MRP from its previous levels. I would typically consider an MRP of 7 percent over the long-bond rate as reasonable based on my review of the relevant academic literature. However, current market conditions—as reflected in elevated bond yield spreads as described above in Section III—suggest that a value of 7.5 percent or even 8.5 percent could be more appropriate at this time. I include two analyses using an MRP of 6.94 and 7.94 percent.[53]

3. Beta

Q64. Can you more fully explain beta?

A64. The basic idea behind beta is that risks that cannot be diversified away in large portfolios matter more than those that can be eliminated by diversification. Beta is a measure of the risks that cannot be eliminated by diversification. That is, it measures the "systematic" risk of a stock—the extent to which a stock's value fluctuates more or less than average when the market fluctuates.

Diversification is a vital concept in the study of risk and return. (Harry Markowitz won a Nobel Prize for work showing just how important it was.) Over the long run, the rate of return on the stock market has a very high standard deviation, on the order of 20 percent per year.[54] Many individual stocks have much higher standard deviations than this. The stock market's standard deviation is "only" about 15-20 percent because when stocks are combined into portfolios, some of the risk of

[53] Duff and Phelps's *Ibbotson SBBI 2017 Valuation Yearbook* reports the realized arithmetic average MRP from 1926 to 2016 to be 6.94 percent.

54 See Brealey, Myers and Allen (2017), *Principles of Corporate Finance, 12th Edition*, McGraw-Hill Irwin, New York, p. 172.

individual stocks is eliminated by diversification. Some stocks go up when others go down, and the average portfolio return—whether positive or negative—is usually less extreme than that of many individual stocks within it. The fact that the market's actual annual standard deviation is so large means that, in practice, the returns on stocks are positively correlated with one another, and to a material degree. The reason is that many factors that make a particular stock go up or down also affect other stocks. Examples include the state of the economy, the balance of trade, and inflation. Thus some risk is "non-diversifiable" in that even a well-diversified portfolio of stocks will experience changes in value caused by these shared risk factors. Single-factor equity risk premium models (such as the CAPM) are based upon the assumption that all of the systematic factors that affect stock returns can be considered simultaneously, through their impact on one factor: the market portfolio. Other models derive somewhat less restrictive conditions under which several factors might be individually relevant.

Again, the basic idea behind all of these models is that risks that cannot be diversified away in large portfolios matter more than those that can be eliminated by diversification, because there are a large number of large portfolios whose managers actively seek the best risk-reward tradeoffs available. (Of course, undiversified investors would like to get a premium for bearing diversifiable risk, but they cannot.)

Q65. What does a particular value of beta signify?

A65. By definition, a stock with a beta equal to 1.0 has average non-diversifiable risk: it goes up or down by 10 percent on average when the market goes up or down by 10 percent. Stocks with betas above 1.0 exaggerate the swings in the market: stocks with betas of 2.0 tend to fall 20 percent when the market falls 10 percent, for example. Stocks with betas below 1.0 are less volatile than the market. A stock with a beta of 0.5 will tend to rise 5 percent when the market rises 10 percent.

Q66. How is beta measured?

A66. The usual approach to calculating beta is a statistical comparison of the sensitivity of a stock's (or a portfolio's) return to the market's return. Many investment services report betas, including Bloomberg and the *Value Line Investment Survey*. Betas are not always calculated in precisely the same way, and therefore must be used with a degree of caution. However, the basic principle that a high beta indicates a risky stock has long been widely accepted by both financial theorists and investment professionals, and is universally reflected in all calculations of beta. *Value Line* calculates betas using five years of weekly return data for a company.[55] In my analyses for these proceedings, I present results using the beta estimates reported by *Value Line*.

Q67. What are the betas that you used for the sample companies?

A67. Table 3 below lists the *Value Line* betas I used to calculate my risk-positioning estimates of the cost of capital for the expanded sample.

Table 3
***Value Line* Betas for the Expanded Sample**

Company	Value Line Betas [1]
Atmos Energy	0.70
Chesapeake Utilities	0.70
ONE Gas Inc.	0.70
South Jersey Inds.	0.85
Southwest Gas	0.80
Spire Inc.	0.70
New Jersey Resources	0.80
Northwest Natural Gas	0.70
WGL Holdings Inc.	0.80
Average	0.75
Subsample Average	0.72

Sources and Notes:
[1]: From Valueline Investment Analyzer as of Jan 8, 2018

[55] *Value Line* Glossary, http://www.valueline.com/Glossary/Glossary.aspx

4. The Empirical CAPM

Q68. What other equity risk premium model do you use?

A68. Empirical research has long shown that the CAPM tends to overstate the actual sensitivity of the cost of capital to beta: low-beta stocks tend to have higher risk premiums than predicted by the CAPM and high-beta stocks tend to have lower risk premiums than predicted. A number of variations on the original CAPM theory have been proposed to explain this finding, but the observation itself can also be used to estimate the cost of capital directly, using beta to measure relative risk by making a direct empirical adjustment to the CAPM.

This second model makes use of these empirical findings. It estimates the cost of capital with the equation,

$$r_S = r_f + \alpha + \beta_S \times (MRP - \alpha) \quad (5)$$

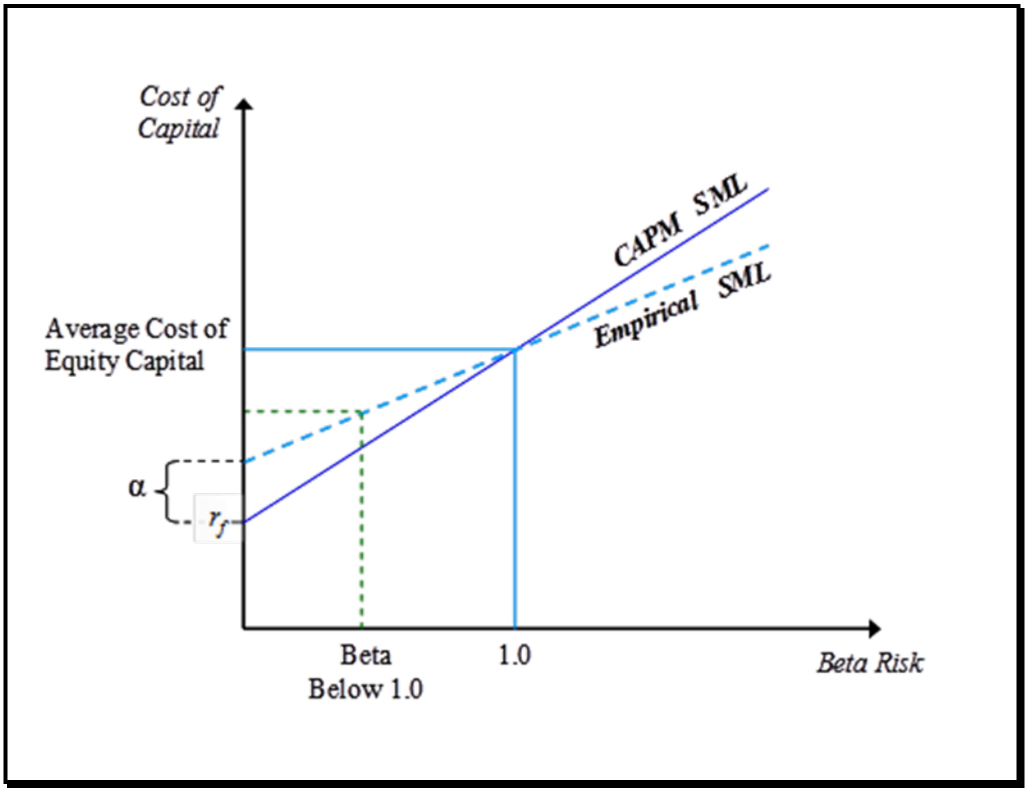
where α is the "alpha" adjustment of the risk-return line, a constant, and the other symbols are defined as for the CAPM (see Equation (4) above).

I label this model the Empirical Capital Asset Pricing Model, or "ECAPM." The alpha adjustment has the effect of increasing the intercept but reducing the slope of the Security Market Line in Figure 1 (page 8), earlier in my testimony which results in a Security Market Line that more closely matches the results of empirical tests. In other words, the ECAPM produces more accurate predictions of eventual realized risk premiums than does the CAPM.

Q69. Why is it appropriate to use the Empirical CAPM?

A69. The CAPM has not generally performed well as an empirical model, but its short-comings are directly addressed by the ECAPM. Specifically, the ECAPM recognizes the consistent empirical observation that the CAPM underestimates (overestimates) the cost of capital for low (high) beta stocks. In other words, the ECAPM is based on recognizing that the actual observed risk-return line is flatter and has a higher intercept than that predicted by the CAPM. The alpha parameter (α) in the ECAPM adjusts for this fact, which has been established by repeated empirical tests of the CAPM. The difference between the CAPM and the type of relationship identified in the empirical studies is depicted in Figure 6 below.

Figure 6
The Empirical Security Market Line



Q70. Does *Value Line* make any adjustments to the beta estimates it reports?

A70. Yes, but *Value Line*'s adjustments are fundamentally different and separate from the ECAPM adjustment I perform. *Value Line*'s adjustments do not correct for the issues raised by the empirical tests of the CAPM. The adjustment to beta corrects the estimate of the relative risk of the company, which is measured along the horizontal axis of the SML. The ECAPM adjusts the risk-return tradeoff (i.e., the slope) in the SML. In other words, the expected return (measured on the vertical axis) for a given level of risk (measured on the horizontal axis) is different from the predictions of the theoretical CAPM. Getting the relative risk of the investment correct does not adjust for the slope of the SML, nor does adjusting the slope correct for errors in the estimation of relative risk.

Q71. Can you explain further why using *Value Line*'s adjusted betas do not correct for the issues raised by empirical tests of the CAPM?

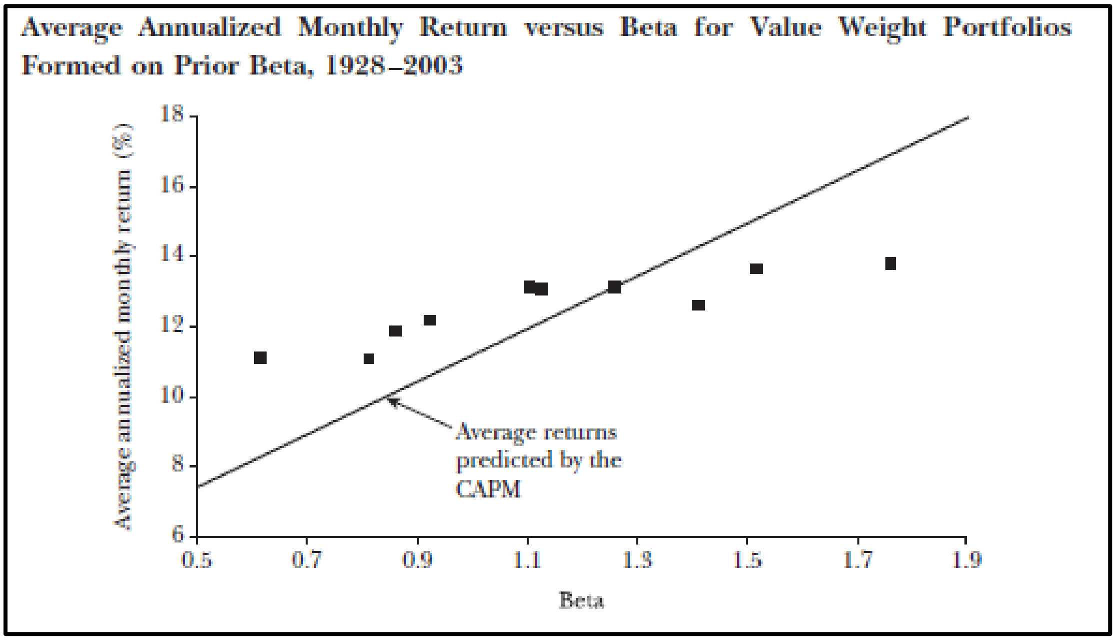
A71. Yes. It is because the issues raised by the empirical tests are completely independent from the reason betas are adjusted. The beta adjustment performed by *Value Line* is based on the method outlined by Professor Marshall Blume,[56] reflecting his empirical observation that historical measurements of a firm's beta are not the best predictors of what that firm's systematic risk *will be* going forward. Professor Blume was able to apply a consistent adjustment procedure to historical betas that increased their accuracy in *forecasting* eventual realized betas. Essentially, Professor Blume's adjustment transforms a historical beta into a better estimate of expected future beta. It is this expected "true" beta that drives investors' expected returns according to the CAPM. Therefore, it is appropriate to use *Value Line's* adjusted betas, rather than raw historical betas, when employing the CAPM to estimate the forward-looking cost of equity capital.

However, the backward-looking empirical tests of the CAPM that gave rise to the ECAPM did not suffer from bias in the measurement of betas. Researchers plotted realized stock portfolio returns against betas measured *over the same time period* to produce plots such as Figure 7 below, which comes from the 2004 paper by Professors Eugene Fama and Kenneth French.[57] The fact that betas and returns were measured contemporaneously means that the betas used in the tests were *already the best possible measure* of the "true" systematic risk over the relevant time period. In other words, no adjustments were needed for these betas. Despite this, researchers observed that the risk-return trade-off predicted by the CAPM was too steep to accurately explain the realized returns. As explained above the ECAPM explicitly corrects for this empirical observation.

56 Blume, Marshall E. (1971), "On the Assessment of Risk," *The Journal of Finance*, 26, pp. 1-10.

57 Fama, Eugene F. & French, Kenneth R, (2004), "The Capital Asset Pricing Model: Theory and Evidence," *Journal of Economic Perspectives, 18(3)*, pp. 25-46.

Figure 7
Evidence from Empirical Tests of the CAPM[58]



Q72. Did the empirical tests that gave rise to the ECAPM use raw betas in their analyses?

A72. They did. However, this is simply because the researchers were able to measure raw betas and realized returns from the same historical period. In other words, no adjustment to the raw beta was necessary to evaluate the market return realized for the same historical period. Hence, the raw betas they measured accurately captured the systematic risk that impacted the returns they measured. In a sense, the measured betas and realized returns were already contemporaneous in the tests of the CAPM that identified the effect shown as illustrated in Figure 6 (page 48) and Figure 7 above.

---

[58] *Ibid.*, p. 33.

Q73. Does the use of adjusted betas in the ECAPM double count the adjustment to the estimated required return on equity?

A73. No. The Blume adjustment to beta and the ECAPM are separate adjustments with no redundancy between them. In fact, both adjustments are necessary to produce the most accurate possible forward-looking estimate of the required return on equity.

A rate of return analyst must use a historical measurement of beta to make a forecast of the expected *future* return on equity. Therefore, the analyst should first apply the Blume adjustment (as *Value Line* does) to get the best estimate of the systematic risk over the (future) period in which (s) he will estimate the ROE. Once the risk measurement is contemporaneous with the returns to be estimated, the analyst should apply the ECAPM to adjust for the empirical shortcomings of the CAPM.

Q74. Can you summarize the independent reasons for using adjusted betas and employing the ECAPM?

A74. Raw historical betas are adjusted to provide a better estimate of *expected* "true" betas, which are the appropriate measure of risk that predicts expected future returns in the CAPM. The ECAPM is used because empirical tests show that *even when the best possible estimate* of "true" beta is used, the CAPM tends to under-predict required returns for low-beta stocks and over-predict required returns for high-beta stocks.

These are independent but complementary adjustments supported by empirical tests of this model of financial theory. Both adjustments are appropriate when using risk-positioning models to estimate the cost of equity.

5. Results from the Risk Positioning Models

Q75. What are the parameters of the scenarios you considered in your risk positioning analyses?

A75. The parameters for the two scenarios are displayed in Table 4 below. The motivation for the scenarios is the empirical observation that the yield spread is higher than normal. The increased yield spread could be the result of an increase in the MRP or downward pressure on the yield of risk-free bonds due to a flight to quality or a combination of the two factors. Therefore, I reduce the risk-free rate for use with a higher estimate of the MRP as illustrated in Table 4. In other words, the approximately 20 bps increase in the yield spread is allocated between an increase in the MRP and the downward pressure on the risk-free rate according to the method described above in Section III. The more of the increase in yield spread that is allocated to the underestimation of the risk-free rate, the less the MRP is increased and vice versa.

Table 4
Risk Positioning Scenario Parameters

Parameters Used in CAPM-based Models		
	Scenario 1	**Scenario 2**
Risk-Free Interest Rate	4.1%	3.9%
Market Equity Risk Premium	6.9%	7.9%

Q76. Can you summarize the results from applying the CAPM and ECAPM methodologies to the sample?

A76. The results of the risk positioning analyses (the CAPM and the ECAPM) are presented in Table 5 below, using *Value Line*'s estimated betas for the expanded sample of companies. (The underlying calculations are also presented in Attachment A.[59]). For the ECAPM, there are two sensitivities: α = 0.5 percent and α = 1.5 percent. The columns display the scenario results for MRP estimates of 6.9 and 7.9 percent in accordance with the adjustments I made to reflect the elevated yield spread as described above. The long-term risk-free interest rate as of January 2018 was 3.94 percent before adjustments for the downward pressure on government yields due to the flight to safety. The ROE estimates in Table 5 reflect the ATWACC and Hamada adjustment procedure estimates adjusted for differences in capital structure between the sample companies and Vectren. Specifically, the ROE associated with each

59 Results for the CAPM and ECAPM based on the ATWACC financial risk adjustment can be found in Attachment A, Schedule No. D5.12 at 49. Results for the CAPM and ECAPM based on the Hamada adjustment can be found in Attachment A, Schedule No. D5.15 at 52-53.

method and a capital structure with 50.6 percent equity is displayed in Table 5 for the *Value Line* betas.

Table 5
Risk Positioning Cost of Equity Estimates

Return on Equity Summary and Sensitivity Analysis U.S. Gas		
Estimated Return on Equity	Scenario 1 [1]	Scenario 2 [2]
Financial Risk Adjusted Method		
CAPM	10.4%	11.1%
ECAPM (α = 0.5%)	10.5%	11.2%
ECAPM (α = 1.5%)	10.8%	11.5%
Hamada Adjustment Without Taxes		
CAPM	10.4%	11.1%
ECAPM (α = 0.5%)	10.4%	11.1%
ECAPM (α = 1.5%)	10.5%	11.2%
Hamada Adjustment With Taxes		
CAPM	10.5%	11.2%
ECAPM (α = 0.5%)	10.5%	11.2%
ECAPM (α = 1.5%)	10.6%	11.3%

Sources and Notes:
Scenario 1: Long-Term Risk Free Rate of 4.14%, Long-Term Market Risk Premium of 6.94%.
Scenario 2: Long-Term Risk Free Rate of 3.94%, Long-Term Market Risk Premium of 7.44%.

Q77. What conclusions do you draw from the risk positioning model (i.e., CAPM and ECAPM) results?

A77. Of the risk positioning estimates, the CAPM values deserve the least weight, because this method does not adjust for the empirical finding that the cost of capital is less sensitive to beta than predicted by the CAPM (which my testimony and exhibits consider by using the ECAPM). Conversely, the ECAPM numbers deserve more weight, because this method adjusts for the empirical findings. The results for Scenario 1 do not fully adjust for the ongoing uncertainty in the capital markets and deserve less weight than the results for Scenario 2 in column [2]. Focusing on the ECAPM (Scenario One) results for the sample, the results range from 10.4 percent to 10.8 percent. The ECAPM risk positioning results for Scenario Two range from 11.1

percent to 11.5 percent. For Scenario 1, the results range from 10.4 percent to 10.8 percent. For Scenario 2, the results range from 11.1 percent to 11.5 percent.

B. Risk Premium Model Estimates

Q78. Did you estimate the cost of equity that results from an analysis of risk premiums implied by allowed ROE's in past utility rate cases?

A78. Yes. In this type of analysis, sometimes called the "risk premium model," the cost of equity capital for utilities is estimated based on the historical relationship between allowed ROE's in utility rate cases and the risk-free rate of interest at the time the ROE's were granted. These estimates add a "risk premium" implied by this relationship to the relevant (prevailing or forecast) risk-free interest rate:

$$Cost\ of\ Equity = r_f + Risk\ Premium \tag{6}$$

Q79. What are the merits of this approach?

A79. First, it estimates the cost of equity from regulated entities as opposed to holding companies, so that the relied upon figure is directly applicable to a rate base. Second, the allowed returns are clearly observable to market participants, who will use this one data input to making investment decisions, so that the information is at the very least a good check on whether the return is comparable to that of other investments. Third, I analyze the spread between the allowed ROE at a given time and the then prevailing interest rate to ensure that I properly consider the interest rate regime at the time the ROE was awarded. This implementation ensures that I can compare allowed ROE granted at different times and under different interest rate regimes.

Q80. How did you use rate case data to estimate the risk premiums for your analysis?

A80. The rate case data from 1990-2017 is derived from Regulatory Research Associates.[60] Using this data I compared (statistically) the average allowed rate of return on equity granted by U.S. state regulatory agencies in natural gas distribution cases to the

---

[60] SNL Financial as of January 31, 2018.

average 20-year Treasury bond yield that prevailed in each quarter.[61] I calculated the allowed utility "risk premium" in each quarter as the difference between allowed returns and the Treasury bond yield, since this represents the compensation for risk allowed by regulators. Then I used the statistical technique of ordinary least squares ("OLS") regression to estimate the parameters of the linear equation:

$$Risk\ Premium = A_0 + A_1 \times (Treausury\ Bond\ Yield) \tag{7}$$

I derived my estimates of A_0 and A_1 using standard statistical methods (OLS regression) and find that the regression has a high degree of explanatory power in a statistical sense (R^2=0.85) and the parameter estimates, A_0 equals 8.407 percent and A_1 equals -0.5611, are statistically significant. The negative slope coefficient reflects the empirical fact that regulators grant smaller risk premiums when risk-free interest rates (as measured by Treasury bond yields) are higher. This is consistent with past observations that the premium investors require to hold equity over government bonds increases as government bond yields decline. In the regression described above the risk premium declined by less than the increase in Treasury bond yields. Therefore, the allowed ROE on average declined by less than 100 basis points when the government bond yield declined by 100 basis points. Based on this analysis, current market conditions suggest an allowed ROE of 10.1 - 10.2 percent for an average risk natural gas LDC.[62]

Q81. What conclusions did you draw from your risk premium analysis?

A81. While the risk premium models based on historical allowed returns are not underpinned by fundamental finance principles in the manner of the CAPM or DCF models, I believe that this analysis, when properly designed and executed and placed in the proper context, can provide useful benchmarks for evaluating whether the

[61] I rely on the 20-year government bond to be consistent with the analysis using the CAPM to avoid confusion about the risk-free rate. While it is important to use a long-term risk-free rate to match the long-lived nature of the assets, the exact maturity is a matter of choice. Rate cases limited to natural gas distribution only (excludes rate cases for transmission or limited-issue rider).

[62] Results for the Risk Premium analysis can be found in Schedule D5.16.

estimated ROE is consistent with recent practice. My risk premium model cost of equity estimates demonstrate that the results of my DCF and CAPM analyses are in line with the allowed return of utility regulators. Because the risk premium analysis as implemented takes into account the interest rate prevailing during the quarter the decision was issued, it provides a useful benchmark for the cost of equity in any interest environment.

C. THE DCF BASED ESTIMATES

Q82. Can you describe the discounted cash flow approach to estimating the cost of equity?

A82. The DCF model takes the first approach to cost of capital estimation described above, i.e., to attempt to estimate the cost of capital in one step instead of estimating the cost of capital for the entire market and then determining the cost of capital for an individual investment. The DCF method assumes that the market price of a stock is equal to the present value of the dividends that its owners expect to receive. The method also assumes that this present value can be calculated by the standard formula for the present value of a cash flow stream:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots + \frac{D_T}{(1+r)^T} \quad (8)$$

where P_0 is the current market price of the stock;
D_t is the dividend cash flow expected at the end of period t;
T is the last period in which a dividend cash flow is to be received; and
r is the cost of equity capital

The formula simply says that the stock price is equal to the sum of the expected future dividends, each discounted for the time and risk between now and the time the dividend is expected to be received.

Most DCF applications go even further, and make strong assumptions that yield a simplification of the standard formula, which then can be rearranged to estimate the cost of capital. Specifically, if investors expect a dividend stream that will grow forever at a steady rate, then the market price of the stock will be given by a very simple formula,

$$P_0 = \frac{D_1}{r-g} \tag{9}$$

where D_1 is the dividend expected at the end of the first period, g is the perpetual growth rate, and P_0 and r are the current market price and the cost of equity capital, as before.

Equation (9) is a simplified version of Equation (8) that can be solved to yield the well-known "DCF formula" for the cost of capital:

$$r = \frac{D_1}{P_0} + g = \frac{D_0}{P_0} \times (1+g) + g \tag{10}$$

where D_0 is the current dividend, which investors expect to increase at rate g by the end of the next period, and the other symbols are defined as before.

Equation (10) says that if Equation (9) holds, the cost of capital equals the expected dividend yield plus the (perpetual) expected future growth rate of dividends. I refer to this as the "simple DCF" model. Of course, the "simple" model is simple because it relies on strong assumptions.[63]

Q83. Are there other versions of the DCF models in addition to the "simple" one?

A83. Yes. One such alternative version is the multistage DCF model. In its "simple" or constant growth rate formulation, the DCF model requires that dividends and earnings grow at a constant rate for companies that earn their cost of capital on average.[64] It is

63 In this context "strong" means assumptions that are unlikely to reflect reality but that also are not expected to have a large effect on the estimate.

64 Why must the two growth rates be equal in a steady-growth DCF model? Think of earnings as divided between reinvestment, which funds future growth, and dividends. If dividends grow faster than earnings, then there is less investment and slower growth each year. Sooner or later dividends will equal earnings. At that point, growth is zero because nothing is being reinvested (dividends are constant). If dividends grow more slowly than earnings, each year a bigger fraction of earnings are reinvested. That makes for ever faster growth. Both scenarios contradict the steady-growth assumption. So if you observe a company with different expectations for dividend and earnings

inconsistent with the theory on which this formulation is based to have varying growth rates in earnings and dividends. If, however, the growth rates for dividends and earnings were expected to vary over some number of years before settling down into a constant growth period, then it would be appropriate to utilize a multistage DCF model. In the multistage model, earnings and dividends can grow at different rates, but must grow at the same rate in the final, constant growth rate period.

Q84. What is your assessment of the DCF model?

A84. The DCF approach is grounded in solid finance theory. It is widely accepted by regulatory commissions and provides useful insight regarding the cost of capital based on forward-looking metrics. DCF estimates of the cost of capital complement those of the CAPM and the ECAPM because the two methods rely on different inputs and assumptions. The DCF method is particularly valuable in the current economic environment, because of the effects on capital market conditions of the Fed's efforts to maintain interest rates at historically low levels which bias the CAPM and ECAPM estimates downward.

However, I recognize that the DCF model, like most models, relies upon assumptions that do not always correspond to reality. For example, the DCF approach assumes that the variant of the present value formula that is used matches the variations in investor expectations for the growth of dividends, and that the growth rate(s) used in that formula match current investor expectations. Less frequently noted conditions, such as the value of real options incorporated in a company's market price, may create issues that the DCF model does not incorporate. Nevertheless, under current economic conditions, because of its forward looking nature, the strengths of the DCF method far outweigh any weaknesses the method may have.

---

growth, you know the company's stock price and its dividend growth forecast are inconsistent with the assumptions of the steady-growth DCF model.

Q85. What growth rate information do you use?

A85. The first step in my DCF analysis (either constant growth or multistage formulations) is to examine a sample of investment analysts' forecasted earnings growth rates from Thomson Reuters IBES and from *Value Line* for companies in the expanded sample.[65] For the long-term growth rate for the final, constant-growth stage of the multistage DCF estimates, I use the most recent long-run GDP growth forecast from Blue Chip Economic Indicators.[66]

Q86. How do these growth rates correspond to the theoretical criteria you discuss above?

A86. The constant-growth formulation of the DCF model, in principle, requires forecasted growth rates, but it is also necessary that the growth rates used go far enough out into the future so that it is reasonable to believe that investors expect a stable growth path afterwards. Under current economic conditions, I believe the forecasted growth rates of investment analysts provide the best available representation of the longer term, steady-state growth rate expectations of investors. Therefore, I feel these growth parameters available to apply to the simple, constant-growth DCF model provide useful estimates of the cost of capital.

Q87. Does the multistage DCF improve upon the simple DCF?

A87. Potentially, but the multistage method assumes a particular smoothing pattern and a long-term growth rate afterwards. These assumptions may not be a more accurate representation of investor expectation than those of the simple DCF. The smoother growth pattern, for example, might not be representative of investor expectations, in which case the multistage model would not increase the accuracy of the estimates. Indeed, amidst uncertainty in capital markets, assuming a simple constant growth rate may be preferable to attempting to model growth patterns in greater detail over

[65] *Value Line* short-term (5 years) EPS growth rates are as of January 8. Thomson Reuters IBES growth rates are as of January 31, 2018. I develop a weighted-average growth rate weighted by the number of analysts and counting *Value Line* as one analyst.

[66] *Blue Chip Economic Indicators*, October 10, 2017.

multiple stages. While it is difficult to determine which set of assumptions comprises a closer approximation of the actual conditions of capital markets, I believe both forms of the DCF model provide useful information about the cost of capital.

Q88. What are the relative strengths and weaknesses of the DCF and risk-positioning methodologies?

A88. Current market conditions affect all cost of capital estimation models to some degree, but the DCF model has at least one advantage over the risk positioning models. Specifically, the DCF model reflects current market conditions more quickly because the market price of a company's stock changes daily. Dividend yields increase when market prices fall and reflect the increased cost of capital. The challenge for the DCF model is that the model requires forecasts of earnings growth rates that are based upon stable economic conditions which are required to satisfy the constant dividend growth rate assumption. Although the dividend yield quickly reacts to changes in the market, the growth rate estimates may be less precise during times of market uncertainty because future growth rates may be more volatile. Nevertheless, because dividend yields and forecast growth rates change quickly, the DCF model is likely to better reflect investors' current cost of capital expectations than the CAPM and ECAPM which relies upon 5 years of historical data.

Q89. What are the DCF estimates for the sample?

A89. The corresponding DCF estimates for the sample are presented in Table 6. For the full sample, the ROE estimate is 13.7 percent for the single-stage "simple DCF" model and 9.4 percent for the multistage model. For the subsample, the ROE estimate is 11.9 percent for the single-stage "simple DCF" model and 9.1 percent for the multistage model.[67]

[67] Calculations and results for the DCF analysis can be found in Schedule D5.5 to Schedule D5.8.

Table 6
DCF Cost of Equity Estimates

Full Sample	
Simple	13.7%
Multi-Stage	9.4%
Subsample	
Simple	11.9%
Multi-Stage	9.1%

I note that the results of the single-stage DCF can be influenced by high individual growth rates.

Q90. What conclusions do you draw from the DCF analysis?

A90. Although I made no adjustment for the current market conditions for the DCF model, the DCF cost of equity estimates are in line with those from the risk positioning models displayed above in Table 6. Specifically, the multistage DCF estimates are lower than the range suggested by the risk positioning analysis while the simple DCF estimates are somewhat higher. At this time, I believe that the DCF estimates indicate that the estimates from Scenario 2 for the risk positioning model are more reliable than those from Scenario 1. Moreover, I believe the forward-looking nature of the DCF model makes the DCF estimates less susceptible to downward biases in inputs that have resulted from the continued uncertainty in the economy and extremely low interest rate environment. Thus I rely more heavily on the DCF estimates than I would in normal economic times.

VI. CONCLUSIONS

Q91. Can you summarize the evidence from the expanded sample regarding the ROE for a natural gas distribution utility of average risk?

A91. Table 5 (page 53) and Table 6 above, summarize the results of the analyses for the risk positioning and DCF models for the sample companies. I also compare these results to the 10.1 – 10.2 percent allowed ROE for an average natural gas LDC suggested by the risk premium model. The results from the CAPM are less reliable than the results from the ECAPM because they do not consider the consistent empirical evidence that the CAPM underestimates the cost of capital for low beta companies, like those in the natural gas LDC sample. Similarly, the results for Scenario 1 are not as reliable as those from Scenario 2 because Scenario 1 ignores the increased MRP resulting from the ongoing uncertainty in the capital markets. As shown in Table 5 (page 53), the ECAPM results range from 10.5 to 11.5 percent. Based on the sample's full cost of capital estimates, which range from 9.1 percent (multi-stage DCF, subsample) to 13.7 percent (simple DCF, full sample), I believe a gas LDC company of average business and financial risk should have an allowed ROE in the range 10 percent to 11 percent.

Q92. What is your recommended range of the ROE for the Company?

A92. As noted above, I judge the Company to be of higher risk than the sample companies on average. I therefore recommend that the Company be allowed an ROE of 10¾ percent, with a range of 10½ to 11 percent, on the equity financed portion of its rate base.

Q93. Why doesn't your recommended range for the samples cover all of the estimates?

A93. I provide an estimate of a reasonable range of required ROE for the sample, and the range of uncertainty is based upon all of the analyses I have done, placing relatively more weight on more reliable methodologies and estimates. I do not try to include all of the resulting estimates in the range because I regard some of the estimates as more reliable than others. For example, the estimates based upon the CAPM are not as reliable as those based upon the ECAPM because the CAPM estimates do not account for the empirical observation that low beta stocks have higher costs of capital than estimated by the CAPM, and high beta stocks have lower costs of capital. Nor is it likely that the lowest estimates in the tables are as reliable as those in the upper end of the range because those estimates do not adequately consider the continued uncertainty in the financial markets.

Q94. Is there any other reason to support an allowed ROE of 10¾ percent?

A94. Yes. It is important to maintain Vectren's access to capital, and maintaining a solid credit rating and outlook is one important aspect to maintaining access to capital. Credit rating agencies are concerned about cash flows. The recent tax reform law will likely put downward pressure on credit ratings for regulated utilities. A supportive allowed return on equity is therefore important to signal an adequate level of stable cash flows and avoid putting downward pressure on Vectren's credit metrics. Maintaining a strong credit rating is particularly critical during a period forecast to have substantial capital investment for infrastructure. In addition, as the Fed continues to adjust its monetary policy, one can expect that the cost of capital will increase although the pace of such an increase cannot be predicted with certainty. This means that estimates at the upper end of the range are more representative of the going-forward cost of capital.

Q95. Does this conclude your prepared direct testimony?

A95. Yes.

QUALIFICATIONS OF MICHAEL J. VILBERT

Dr. Michael J. Vilbert is a Principal in the The Brattle Group's San Francisco office and has more than 20 years of experience as an economic consultant. He is an expert in cost of capital, financial planning and valuation who has advised clients on these matters in the context of a wide variety of investment and regulatory decisions. In the area of regulatory economics, he has testified or submitted testimony on the cost of capital for regulated companies in the water, electric, natural gas and petroleum industries in the U.S. and Canada. His testimony has addressed the effect of regulatory policies such as decoupling or must-run generation on a regulated company's cost of capital and the appropriate way to estimate the cost of capital for companies organized as Master Limited Partnerships. He analyzed issues associated with situations imposing asymmetric risk on utilities, the prudence of purchased power contracts, the economics of energy conservation programs, the appropriate incentives for investment in electric transmission assets and the effect of long-term purchased power agreements on the financial risk of a company. He has served as a neutral arbitrator in a contract dispute and analyzed the effectiveness of a company's electric power supply auction. He has also estimated economic damages and analyzed the business purpose and economic substance of tax related transactions, valued assets in arbitration for purchase at the end of the contract, estimated the stranded costs of resulting from the deregulation of electric generation and from the municipalization of an electric utility's distribution assets and addressed the appropriate regulatory accounting for depreciation and goodwill.

He received his Ph.D. in Financial Economics from the Wharton School of the University of Pennsylvania, an MBA from the University of Utah, an M.S. from the Fletcher School of Law and Diplomacy, Tufts University, and a B.S. degree from the United States Air Force Academy. He joined The Brattle Group in 1994 after a career as an Air Force officer, where he served as a fighter pilot, intelligence officer, and professor of finance at the Air Force Academy.

REPRESENTATIVE CONSULTING EXPERIENCE

- Dr. Vilbert served as the consulting expert in several cases for the U.S. Department of Justice and the Internal Revenue Service regarding the business purpose and economic substance of a series of tax related transactions. These projects required the analysis of a complex series of financial transactions including the review of voluminous documentary evidence and required expertise in financial theory, financial market as well as accounting and financial statement analysis.

- In a securities fraud case, Dr. Vilbert designed and created a model to value the private placement stock of a drug store chain as if there had been full disclosure of the actual financial condition of the firm. He analyzed key financial data and security analysts’= reports regarding the future of the industry in order to recreate pro forma balance sheet and income statements under a variety of scenarios designed to establish the value of the firm.

- For pharmaceutical companies rebutting price-fixing claims in antitrust litigation, Dr. Vilbert was a member of a team that prepared a comprehensive analysis of industry profitability. The analysis replicated, tested and critiqued the major recent analyses of drug costs, risks and returns. The analyses helped develop expert witness testimony to rebut allegations of excess profits.

- For an independent electric power producer, Dr. Vilbert created a model that analyzed the reasonableness of rates and costs filed by a natural gas pipeline. The model not only duplicated the pipeline=s rates, but it also allowed simulation of a variety of Awhat if@ scenarios associated with cost recovery under alternative time patterns and joint cost allocations. Results of the analysis were adopted by the intervenor group for negotiation with the pipeline.

- For the CFO of an electric utility, Dr. Vilbert developed the valuation model used to support a stranded cost estimation filing. The case involved a conflict between two utilities over the responsibility for out-of-market costs associated with a power purchase contract between them. In addition, he advised and analyzed cost recovery mechanisms that would allow full recovery of the stranded costs while providing a rate reduction for the company=s rate payers.

- Dr. Vilbert has testified as well as assisted in the preparation of testimony and the development of estimation models in numerous cost-of-capital cases for natural gas pipeline, water utility and electric utility clients before the Federal Energy Regulatory Commission (AFERC@) and state regulatory commissions. These have spanned standard estimation techniques (e.g., Discounted Cash Flow and Risk Positioning models). He has also developed and applied more advanced models specific to the industries or lines of business in question, e.g., based on the structure and risk characteristics of cash flows, or based on multi-factor models that better characterize regulated industries.

- Dr. Vilbert has valued several large, residual oil-fired generating stations to evaluate the possible conversion to natural gas or other fuels. In these analyses, the expected pre- and post-conversion station values were computed using a range of market electricity and fuel cost conditions.

- For a major western electric utility, Dr. Vilbert helped prepare testimony that analyzed the prudence of QF contract enforcement. The testimony demonstrated that the utility had not been compensated in its allowed cost of capital for major disallowances stemming from QF contract management.

- Dr. Vilbert analyzed the economic need for a major natural gas pipeline expansion to the Midwest. This involved evaluating forecasts of natural gas use in various regions of the United States and the effect of additional supplies on the pattern of natural gas pipeline use. The analysis was used to justify the expansion before the FERC and the National Energy Board of Canada.

- For a Public Utility Commission in the Northeast, Dr. Vilbert analyzed the auction of an electric utility=s purchase power agreements to determine whether the outcome of the auction was in the ratepayers= interest. The work involved the analysis of the auction procedures as well as the benefits to ratepayers of transferring risk of the PPA payments to the buyer.

- Dr. Vilbert led a team tasked to determine whether bridge tolls were "just and reasonable" for a non-profit port authority. Determination of the cost of service for the authority required estimation of the value of the authority's assets using the trended original cost methodology as well as evaluation of the operations and maintenance budgets. Investment costs, bridge traffic information and inflation indices covering a 75 year period were utilized to estimate the value of four bridges and a passenger transit line valued in excess of $1 billion.

- Dr. Vilbert helped a recently privatized railroad in Brazil develop an estimate of its revenue requirements, including a determination of the railroad=s cost of capital. He also helped evaluate alternative rate structures designed to provide economic incentives to shippers as well as to the railroad for improved service. This involved the explanation and analysis of the contribution margin of numerous shipper products, improved cost analysis and evaluation of bottlenecks in the system.

- For a utility in the Southeast, Dr. Vilbert quantified the company=s stranded costs under several legislative electric restructuring scenarios. This involved the evaluation of all of the company=s fossil and nuclear generating units, its contracts with Qualifying Facilities and the prudence of those QF contracts. He provided analysis concerning the impact of securitizing the company=s stranded costs as a means of reducing the cost to the ratepayers and several alternative designs for recovering stranded costs.

- For a recently privatized electric utility in Australia, Dr. Vilbert evaluated the proposed regulatory scheme of the Australian Competition and Consumer Commission for the company=s electric transmission system. The evaluation highlighted the elements of the proposed regulation which would impose uncompensated asymmetric risks on the company and the need to either eliminate the asymmetry in risk or provide additional compensation so that the company could expect to earn its cost of capital.

- For an electric utility in the Southwest, Dr. Vilbert helped design and create a model to estimate the stranded costs of the company=s portfolio of Qualifying Facilities and Power Purchase contracts. This exercise was complicated by the many variations in the provisions of the contracts that required modeling in order to capture the effect of changes in either the performance of the plants or in the estimated market price of electricity.

- Dr. Vilbert helped prepare the testimony responding to a FERC request for further comments on the appropriate return on equity for electric transmission facilities. In addition, Dr. Vilbert was a member of the team that made a presentation to the FERC staff on the expected risks of the unbundled electric transmission line of business.

- Dr. Vilbert and Mr. Frank C. Graves, also of The Brattle Group, prepared testimony evaluating an innovative Canadian stranded cost recovery procedure involving the auctioning of the output of the province=s electric generation plants instead of the plants themselves. The evaluation required the analysis of the terms and conditions of the long-term contracts specifying the revenue requirements of the plants for their entire forecasted remaining economic life and required an estimate of the cost of capital for the plant owners under this new stranded cost recovery concept.

- Dr. Vilbert served as the neutral arbitrator for the valuation of a petroleum products tanker. The valuation required analysis of the Jones Act tanker market and the supply and demand balance of the available U.S. constructed tanker fleet.

- Dr. Vilbert evaluated the appropriate Abareboat@ charter rate for an oil drilling platform for the renewal period following the end of a long-term lease. The evaluation required analysis of the market for oil drilling platforms around the world including trends in construction and labor costs and the demand for platforms in varying geographical environments.

- Dr. Vilbert and Dr. Villadsen, also of The Brattle Group, evaluated the offer to purchase the assets of Pentex Alaska Natural Gas Company, LLC on behalf of the Western Finance Group for presentation to the Board of the Alaska Industrial Development and Export Authority. The report compared the proposed purchase price with selected trading and transaction multiples of comparable companies.

PRESENTATIONS

"Moving Toward Value in Utility Compensation – Shareholder Value Concept," with A. Lawrence Kolbe, California PUC Workshop, June 13, 2016.

"Natural Gas Pipeline FERC ROE," INGAA Rate of Return Seminar, with Mike Tolleth, March 23, 2016.

"The Cost of Capital for Alabama Power Company," Public Service Commission public meeting, July 17, 2013.

"An Empirical Study of the Impact of Decoupling on the Cost of Capital," Center for Research in Regulated Industries, Shawnee on Delaware, PA, May 17, 2013.

“Point – Counterpoint: The Regulatory Compact and Pipeline Competition,” with (Jonathan Lesser, Continental Economics), Energy Bar Association, Western Meeting, February 22, 2013

“Introduction to Retail Rates,” presented to California Water Services Company, 18-19 November 2010.

“Impact of the Ongoing Economic Crisis on the Cost of Capital of the U.S. Utility Sector”, National Association of Water Companies: New York Chapter, Albany, NY, May 21, 2009.

“Impact of the Ongoing Economic Crisis on the Cost of Capital of the U.S. Utility Sector”, New York Public Service Commission, Albany, NY, April 20, 2009.

ACurrent Issues in Explaining the Cost of Capital to Utility Commissions@ Cost of Capital Seminar, Philadelphia, PA, 2008.

ARevisiting the Development of Proxy Groups and Relative Risk Analysis,@ Society of Utility and Regulatory Financial Analysts: 39th Financial Forum, April 2007.

ACurrent Issues in Estimating the Cost of Capital,@ *EEI Electric Rates Advanced Course*, Madison, WI, 2006, 2007, 2008, 2009, 2010 and 2011.

ACurrent Issues in Cost of Capital,@ with Bente Villadsen, *EEI Electric Rates Advanced Course*, Madison, WI, 2005.

ACost of Capital - Explaining to the Commission - Different ROEs for Different Parts of the Business,@ *EEI Economic Regulation & Competition Analysts Meeting*, May 2, 2005.

ACost of Capital Estimation: Issues and Answers,@ *MidAmerican Regulatory Finance Conference*, Des Moines, IA, April 7, 2005.

AUtility Distribution Cost of Capital,@ *EEI Electric Rates Advanced Course*, Madison, WI, July 2004.

ANot Your Father=s Rate of Return Methodology,@ *Utility Commissioners/Wall Street Dialogue*, NY, May 2004.

AIssues for Cost of Capital Estimation,@ with Bente Villadsen, *Edison Electric Institute Cost of Capital Conference*, Chicago, IL, February 2004.

AUtility Distribution Cost of Capital,@ *EEI Electric Rates Advanced Course*, Bloomington, IN, 2002, 2003.

PUBLICATIONS

Risk and Return for Regulated Industries, The Brattle Group, Bente Villadsen, Michael J. Vilbert, Dan Harris, and A. Lawrence Kolbe, Elsevier Academic Press, Cambridge, MA, 2017.

"Effect on the Cost of Capital of Ratemaking that Relaxes the Linkage between Revenue and kWh Sales: An Updated Empirical Investigation of the Electric Industry," Michael J. Vilbert, Joseph B. Wharton, Shirley Zhang, and James Hall, *The Brattle Group*, November 2016.

"Decoupling and the Cost of Capital," Joe Wharton and Michael Vilbert, *The Electricity Journal, Volume 28, Issue 7,* August/September 2015.

"The Impact of Revenue Decoupling on the Cost of Capital for Electric Utilities: An Empirical Investigation," prepared for The Energy Foundation by Michael J. Vilbert, Joseph B. Wharton, Charles Gibbons, Melanie Rosenberg, and Yang Wei Neo, March 20, 2014.

"Estimating the Cost of Equity for Regulated Companies," (with P.R. Carpenter, Bente Villadsen, T. Brown, and P. Kumar), prepared for the Australian Pipeline Industry Association and filed with the Australian Energy Regulator and the Economic Regulation Authority, Western Australia, February 2013.

"Survey of Cost of Capital Practices in Canada," (with Bente Villadsen and Toby Brown), prepared for British Columbia Utilities Commission, May 2012.

"Impact of Portland Harbor Remediation Costs on City of Portland Water and Sewer Rates," with Professor David Sunding, March 2012.

"The Impact of Decoupling on the Cost of Capital – An Empirical Study," Joseph B. Wharton, Michael J. Vilbert, Richard E. Goldberg, and Toby Brown, Discussion Paper, *The Brattle Group*, March 2011, revised July 2012.

"Review of Regulatory Cost of Capital Methodologies," (with Bente Villadsen and Matthew Aharonian), Canadian Transportation Agency, September 2010.

"Understanding Debt Imputation Issues,@ by Michael J. Vilbert, Bente Villadsen and Joseph B. Wharton, *Edison Electric Institute*, June 2008.

"Measuring Return on Equity Correctly: Why current estimation models set allowed ROE too low," by A. Lawrence Kolbe, Michael J. Vilbert and Bente Villadsen, *Public Utilities Fortnightly*, August 2005.

"The Effect of Debt on the Cost of Equity in a Regulatory Setting," by A. Lawrence Kolbe, Michael J. Vilbert, Bente Villadsen and The Brattle Group, *Edison Electric Institute*, April 2005.

"Flaws in the Proposed IRS Rule to Reinstate Amortization of Deferred Tax Balances Associated with Generation Assets Reorganized in Industry Restructuring," by Frank C. Graves and Michael J. Vilbert, white paper for *Edison Electric Institute* (EEI) to the IRS, July 25, 2003.

TESTIMONY

Direct testimony before the Public Utilities Commission of the State of Hawaiʻi on behalf of Young Brothers, Limited, Docket No. 2017-0363, on the cost of capital for Young Brothers regulated intrastate barge operations, March 2018.

Direct testimony before the Michigan Public Service Commission on behalf of the DTE Gas Company, Case No. U-18999, on the cost of common equity capital for DTE Gas Company's regulated natural gas distribution assets, February 2018.

Supplemental testimony before the Public Utilities Commission of the State of Hawaiʻi on behalf of Hawaiian Electric Company, Inc., Docket No. 2016-0328, with regard to the effect on the cost of capital of decoupling ratemaking that relaxes the linkage between revenue and kWh sales, February 2018.

Direct testimony before the Public Utilities Commission of the State of Hawaiʻi on behalf of Maui Electric Company, Limited, Docket No. 2017-0150, with regard to the effect on the cost of capital of decoupling ratemaking that relaxes the linkage between revenue and kWh sales, October 2017.

Rebuttal testimony before the California Public Utilities Commission on behalf of California-American Water Company, Application 15-07-019, Phase 3A and Phase 3b, on the economic effect on the Company and the applicability of a fine based upon California-American Water Company's administration of its tariff for the Monterey Water District, August 2017.

Direct and rebuttal testimony before the Corporation Commission of Oklahoma on behalf of Public Service Company of Oklahoma, Cause No. PUD201700151, on the cost of capital for Public Service Company of Oklahoma's regulated assets, June 2017 and October 2017.

Direct and rebuttal testimony before the California Public Utilities Commission on behalf of California Water Services Company, Application No. A.1704-006, on the cost of capital for California Water Services Company's regulated assets, April 2017 and August 2017.

Direct and rebuttal testimony before the Michigan Public Service Commission on behalf of the DTE Electric Company, (Case No. U-18255) on the cost of common equity capital for DTE Electric's regulated electric assets, April 2017 and September 2017.

Prepared direct testimony before the Federal Energy Regulatory Commission, Docket No. RP17-598-000 on behalf of Great Lakes Gas Transmission Limited Partnership, regarding the appropriate ROE to allow for its regulated natural gas pipeline assets, March 2017.

Prepared direct testimony before the North Carolina Utilities Commission, Docket No. G-39, Sub 38, on behalf of the Cardinal Pipeline Company, LLC regarding the appropriate allowed ROE for the Company's pipeline assets, March 2017.

Prepared direct testimony before the Federal Energy Regulatory Commission, Docket No. ER17-706-000 on behalf of Gridliance West Transco LLC, regarding Gridliance West's application pursuant to section 205 of the Federal Power Act regarding the appropriate ROE, cost of debt, and capital structure to allow Gridliance West Transco LLC to earn on the transmission facilities acquired from Valley Electric Association, December 2016.

Prepared direct testimony and supporting exhibits before the Federal Energy Regulatory Commission, Docket No. EC17-049-000, on behalf of Gridliance West Transco LLC, regarding GridLiance West's application pursuant to section 203 of the Federal Power Act (FPA) to acquire certain high voltage transmission facilities from Valley Electric Transmission Association, LLC (VETA) through its parent non-profit electric cooperative parent Valley Electric Association, Inc. (Valley Electric), December 2016.

Prepared direct testimony and supporting exhibits before the Federal Energy Regulatory Commission, Docket No. ER16-2632-000, on behalf of Trans Bay Cable LLC, regarding the appropriate ROE and capital structure to allow for its regulated electric transmission assets, September 2016.

Prepared direct and rebuttal testimony before the Public Utilities Commission of Hawai‘i on the effect on the cost of capital of decoupling ratemaking that relaxes the linkage between revenue and kWh sales on behalf of Hawai‘i Electric Light Company, Inc. Docket No. 2015-0170, August 2016 and June 2017.

Direct testimony before the Michigan Public Service Commission on behalf of the Detroit Thermal, LLC (Case No. U-18131) on the cost of common equity capital for Detroit Thermal's regulated steam service, July 2016.

Pre-filed direct testimony and supporting exhibits before the Rhode Island Public Utilities Commission on behalf of The Narragansett Electric Company d/b/a National Grid Docket No. 47xx regarding Petition for the Approval of Gas Capacity Contracts and Cost Recovery, June 2016.

Prepared direct testimony and supporting exhibits before the Federal Energy Regulatory Commission, Docket No. RP16-440-000, on behalf of ANR Pipeline Company, regarding the appropriate ROE to allow for its regulated natural gas pipeline assets, January 2016.

Pre-filed direct testimony before the Massachusetts Department of Public Utilities on behalf of Massachusetts Electric Company and Nantucket Electric Company d/b/a National Grid regarding the risk transfer inherent in signing long-term contracts for natural gas pipeline capacity, Docket No. D.P.U. 16-05, January 2016.

Direct and rebuttal testimony before the Michigan Public Service Commission on behalf of the DTE Electric Company (Case No. U-18014) on the cost of capital for DTE Electric Company's regulated electric assets, January 2016 and July 2016.

Rebuttal testimony before the Public Utility Commission of Texas on behalf of Ovation Acquisition I, L.L.C., Ovation Acquisition II, L.L.C., and Shary Holdings, L.L.C. concerning the adequacy of Oncor Electric Distribution Company's (Oncor) liquidity, access to capital and financial risk with regard to the proposed restructuring of Oncor, PUC Docket No. 451888, December, 2015.

Direct and rebuttal testimony before the Michigan Public Service Commission on behalf of the DTE Gas Company (Case No. U-17799) on the cost of capital for DTE Gas Company's natural gas distribution assets, December 2015 and May 2016.

Prepared direct testimony before the Federal Energy Regulatory Commission, Docket No. ER15-2594-000, on behalf of South Central MCN, LLC, regarding the appropriate ROE to include in the transmission rate formula (Formula Rate) to establish an annual transmission revenue requirement (ATRR) for transmission service over facilities that SCMCN will own in the Southwest Power Pool, Inc. (SPP) region, September 2015.

"Report on Gas LDC multiples," with Bente Villadsen, *Alaska Industrial Development and Export Authority*, May 2015.

Direct and reply testimony before the Regulatory Commission of Alaska on behalf of Cook Inlet Natural Gas Storage Alaska, LLC, Docket No. U-15-016 on the appropriate allocation of the proceeds from the sale of excess Found Native Gas discovered incidental to the construction of the storage facility, April 2015 and July 2015.

Direct testimony before the Michigan Public Service Commission on behalf of the Detroit Edison Electric Company (Case No. U-17767) on the cost of capital for DTE's electric utility assets, December 2014.

Direct and rebuttal testimony before the Washington Utilities and Transportation Commission on behalf of Puget Sound Energy, Inc. Docket Nos. UE-130137 and UG-130138 (consolidated) remand proceeding with regard to the effect of decoupling on the cost of capital, November 2014 and December 2014.

Initial and Reply Statement of Position before the Public Utilities Commission of Hawaiʻi In the Matter of Instituting an Investigation to Reexamine the Existing Decoupling Mechanisms for Hawaiian Electric Company, Inc., Hawaiʻi Electric Light Company, Inc., and Maui Electric Company, Limited, Docket No. 2013-0141, with Dr. Toby Brown and Dr. Joseph B. Wharton, May 2014 and September 2014.

Direct and rebuttal testimony before the Pennsylvania Public Utility Commission on behalf of Metropolitan Edison Company (Docket No. R-2014-2428745), Pennsylvania Electric Company (Docket No. R-2014-2428743), Pennsylvania Power Company (Docket No. R-2014-2428744), and West Penn Power Company (Docket No. R-2014-2428742) regarding the appropriate cost of common equity for the companies, September 2014 and December 2014.

Direct and rebuttal testimony before the Public Service Commission of West Virginia in the Matter of the Application of Monongahela Power Company and The Potomac Edison Company, Case No. 14-0702-E-42T for approval of a general change in rates and tariffs, June 2014 and October 2014.

Direct testimony before the Public Utilities Commission of Ohio in the Matter of the Determination of the Existence of Significantly Excessive Earnings for 2012 Under the Electric Security Plans of Ohio on behalf of the Edison Company, The Cleveland Electric Illuminating Company, and The Toledo Edison Company, Case No. 14-0828-EL-UNC, May 2014.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER14-1332-000, on behalf of DATC Path 15, LLC, regarding the appropriate ROE to include in the Submission of Revisions to Appendix I in TO Tariff Reflecting Updated TRR to be Effective February, 2014.

Direct testimony, rebuttal testimony and sur-surrebuttal testimony before the Arkansas Public Service Commission regarding the appropriate ROE to allow In the Matter of the Application of SourceGas Arkansas Inc., Docket No. 13-079-U for Approval of a General Change in Rates, and Tariffs, September 2013, March 2014, and April 2014.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER13-2412-000, on behalf of Trans Bay Cable LLC, regarding the appropriate ROE to include in the Submission of Revisions to Appendix I of the Trans Bay Transmission Owner Tariff to be Effective 11/23/2013, September 2013.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER13-2412-000, on behalf of Trans Bay Cable LLC, regarding the appropriate ROE to include in the Submission of Revisions to Appendix I of the Trans Bay Transmission Owner Tariff to be Effective 11/23/2013, September 2013.

Presentation on behalf of Alabama Power Company with regard to the appropriate cost of capital for the Rate Stabilization and Equalization mechanism, Dockets 18117 and 18416, July 2013.

Direct testimony before the Public Utilities Commission of Ohio in the Matter of the Determination of the Existence of Significantly Excessive Earnings for 2012 Under the Electric Security Plans of Ohio on behalf of the Edison Company, The Cleveland Electric Illuminating Company, and The Toledo Edison Company, Case No. 13-1147-EL-UNC, May 2013.

Expert Report, with A. Lawrence Kolbe and Bente Villadsen, on cost of equity, non-recovery of operating cost and asset retirement obligations on behalf of the behalf of oil pipeline in arbitration, April 2013.

Direct and Rebuttal testimony before the Public Utilities Commission of the State of Colorado on behalf of Rocky Mountain Natural Gas LLC regarding the cost of capital for an intrastate natural gas pipeline, Docket No. 13AL-143G, with Advice Letter No. 77, January 2013 and October 2013.

Rebuttal Testimony before the Public Utilities Commission of the State of California on behalf of Southern California Edison regarding Application 12-04-015 of Southern California Edison Company (U 338-E) For Authority to Establish Its Authorized Cost of Capital for Utility Operations for 2013 and to Reset the Annual Cost of Capital Adjustment Mechanism , August 2012.

Direct testimony and supporting exhibits on behalf of Transcontinental Gas Pipeline Company, LLC, before the Federal Energy Regulatory Commission, on the Cost of Capital for Interstate Natural Gas Pipeline assets, Docket No. RP12-993-000, August 2012.

Direct Testimony before the North Carolina Utilities Commission on behalf of Cardinal Pipeline Company LLC, regarding the cost of capital for an intrastate natural gas pipeline, Docket G-39, Sub 28, August 2012.

Joint Rebuttal Testimony before the California Public Utility Commission on behalf of California American Water Company, regarding Application of California-American Water Company (U210W) for Authorization to increase its Revenues for Water Service, Application 10-07-007, and In the Matter of the Application of California-American Water Company (U210W) for an Order Authorizing and Imposing a Moratorium on New Water Service Connections in its Larkfield District, Application 11-09-016, August 2012.

Direct testimony before the Public Utilities Commission of Ohio, In the Matter of the Determination of the Existence of Significantly Excessive Earnings for 2011 Under the Electric Security Plan of Ohio Edison Company, The Cleveland Electric Illuminating Company, and The Toledo Edison Company, Case No. 12-1544-EL-UNC, May 2012.

Deposition testimony in *Tahoe City Public Utility District, Plaintiff vs. Case No. SCV 27283 Tahoe Park Water Company, Lake Forest Water Company, Defendants*, May 2012.

Deposition testimony in *Primex Farms, LLC, Plaintiff, v. Roll International Corporation, Westside Mutual Water Company, LLC, Paramount Farming Company, LLC, Defendants*, April 2012.

Direct and rebuttal testimony before the Michigan Public Service Commission, Case No. U-16999, on behalf of Michigan Consolidated Gas Company, regarding cost of service for natural gas distribution assets, April 2012 and October 2012.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. PA10-13-000, on behalf of ITC Holdings Corp. regarding a rehearing for FERC Staff, Office of Enforcement, Division of Audits, Report on the appropriate accounting for goodwill for the acquisition of ITC Midwest assets from Interstate Power and Light Company, February 2012.

Rebuttal testimony before the Florida Public Service Commission, Docket No. 110138-EL, on behalf of Gulf Power, a Southern Company, on the method to adjust the return on equity for differences in financial risk, November 2011.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER12-296-000, on behalf of Public Service Electric and Gas Company on the Cost of Capital and for Incentive Rate Treatment for the Northeast Grid Reliability Transmission Project, October 2011.

Rebuttal Evidence before the National Energy Board in the matter of AltaGas Utilities Inc., 2010-2012 GRA Phase I, Application No. 1606694; Proceeding I.D. 904, October, 2011.

Report before the Arbitrator on behalf of Canadian National Railway Company in the matter of a Submission by Tolko Marketing and Sales LTD for Final Offer Arbitration of the Freight Rates and Conditions Associated with Respect to the Movement of Lumber by Canadian National Railway Company from High Level, Alberta to Various Destinations in the Vancouver, British Columbia Area, October, 2011.

Written direct and reply evidence before the National Energy Board in the matter of the National Energy Board Act, R.S.C. 1985, c. NB7, as amended, and the Regulations made thereunder; and in the matter of an application by TransCanada PipeLines Limited for orders pursuant to Part I and Part IV of the *National Energy Board Act*, for determining the overall fair return on capital in the business and services restructuring and Mainline 2012 – 2013 toll application, RH-003-2011, September 2011 and May 2012.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. PA10-13-000, on behalf of ITC Holdings Corp. in response to FERC Staff, Office of Enforcement, Division of Audits, Draft Report on the appropriate accounting for goodwill for the acquisition of ITC Midwest assets from Interstate Power and Light Company, July 2011.

Initial testimony before the Public Utilities Commission of Ohio, Case No. 11-4553-EL-UNC, In the Matter of the Determination of the Existence of Significantly Excessive Earnings for 2010 Under the Electric Security Plan of Ohio Edison Company, The Cleveland Electric Illuminating Company, and The Toledo Edison Company, July 2011.

Rebuttal testimony before the Public Utilities Commission of the State of California, Docket No. A.10-09-018, on behalf of California American Water Company, on Application of California American Water Company (U210W) for Authorization to Implement the Carmel River Reroute and San Clemente Dam Removal Project and to Recover the Costs Associated with the Project in Rates, June 2011.

Direct and rebuttal testimony before the Public Utilities Commission of the State of California, Docket No. A.11-05-001, on behalf of California Water Service Company, on the Cost of Capital for Water Distribution Assets, April 2011 and September 2011.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER11-013-000, on behalf of the Atlantic Wind Connection Companies, on the Cost of Capital and Cost of Capital incentive adders for Electric Transmission Assets, December 2010.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. RP11-1566-000, on behalf Tennessee Gas Pipeline Company, on the Cost of Capital for Natural Gas Transmission Assets, November 2010.

Direct and rebuttal testimony before the Michigan Public Service Commission, In the matter of the application of The Detroit Edison Company, for authority to increase its rates, amend its rate schedules and rules governing the distribution and supply of electric energy, and for miscellaneous accounting authority, Case No. U-16472, October 2010 and April 2011.

Direct and rebuttal testimony before the Federal Energy Regulatory Commission, Docket No. RP10-1398-000, on behalf of El Paso Natural Gas Company, on the Cost of Capital for Natural Gas Transmission Assets, September 2010 and September 2011.

Direct testimony before the Public Utilities Commission of Ohio, Case No. 10-1265-EL-UNC, In the Matter of the Determination of the Existence of Significantly Excessive Earnings for 2009 Under the Electric Security Plan of Ohio Edison Company, The Cleveland Electric Illuminating Company, and The Toledo Edison Company, September 2010.

Direct testimony before the Michigan Public Service Commission, Case No. U-16400, on behalf of Michigan Consolidated Gas Company, regarding cost of service for natural gas distribution assets, July 15, 2010.

Direct testimony before the Oklahoma Corporation Commission, Cause No. PUD 201000050, on behalf of Public Service Company of Oklahoma, regarding cost of service for a regulated electric utility, June 2010.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER10-516-000, on behalf of South Caroline Gas and Electric Company, on the Cost of Capital for Electric Transmission Assets, December 2009.

Direct and Rebuttal Testimony before the California Public Utilities Commission regarding cost of service for San Joaquin Valley crude oil pipeline on behalf of Chevron Products Company, Docket Nos. A.08-09-024, C.08-03-021, C.09-02-007 and C.09-03-027, December 2009 and April 2010.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER10-159-000, on behalf of Public Service Electric and Gas Company, on the incentive Cost of Capital for the Branchburg-Roseland-Hudson 500 kV Line electric transmission project ("BRH Project"), October 2009.

Rebuttal testimony before the Florida Public Service Commission in re: Petition for Increase in Rates by Progress Energy Florida, Inc., Docket No. 090079-EI, August 2009.

Direct and rebuttal testimony before the State of New Jersey Board of Public Utilities in the Matter of the Petition of Public Service Electric and Gas Company for Approval of an Increase in Electric and Gas Rates and for Changes in the Tariffs for Electric and Gas Service, B.P.U.N.J.

No. 14 Electric and B.P.U.N.J No. 14 Gas Pursuant to N.J.S.A. 48:2-21 and N.J.S.A. 48:2-21.1 and for Approval of a Gas Weather Normalization Clause; a Pension Expense Tracker and for other Appropriate Relief BPU Docket No. GR09050422, June 2009 and December 2009.

Direct and rebuttal testimony before the Public Service Commission of Wisconsin, Docket No. 6680-UR-117, on behalf of Wisconsin Power and Light Company, on the cost of capital for electric and natural gas distribution assets, May 2009 and September 2009.

Written evidence before the Régie de l'Énergie on behalf of Gaz Métro Limited Partnership, Cause Tarifaire 2010, R-3690-2009, on the Cost of Capital for natural gas transmission assets, May 2009.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER09-681-000, on behalf of Green Power Express, LLP, on the Cost of Capital for Electric Transmission Assets, February 2009.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER09-548-000, on behalf of ITC Great Plains, LLC, on the Cost of Capital for Electric Transmission Assets, January 2009.

Written and Reply Evidence before the Alberta Utilities Commission in the matter of the Alberta Utilities Commission Act, S.A. 2007, c. A-37.2, as amended, and the regulations made thereunder; and IN THE MATTER OF the Gas Utilities Act, R.S.A. 2000, c. G-5, as amended, and the regulations made thereunder; and IN THE MATTER OF the Public Utilities Act, R.S.A. 2000, c. P-45, as amended, and the regulations made thereunder; and IN THE MATTER OF Alberta Utilities Commission 2009 Generic Cost of Capital Hearing, Application No. 1578571/Proceeding No. 85. 2009 Generic Cost of Capital Proceeding on behalf of AltaGas Utilities Inc., November 2008 and May 2009.

Written Evidence before the Alberta Utilities Commission in the matter of the Alberta Utilities Commission Act, S.A. 2007, c. A-37.2, as amended, and the regulations made thereunder; and IN THE MATTER OF the Gas Utilities Act, R.S.A. 2000, c. G-5, as amended, and the regulations made thereunder; and IN THE MATTER OF the Public Utilities Act, R.S.A. 2000, c. P-45, as amended, and the regulations made thereunder; and IN THE MATTER OF Alberta Utilities Commission 2009 Generic Cost of Capital Hearing, Application No. 1578571/Proceeding No. 85. 2009 Generic Cost of Capital Proceeding on behalf of NGTL, November 2008.

Direct and rebuttal testimony before the Public Service Commission of West Virginia, Case No. 08-1783-G-PC, on behalf of Dominion Hope Gas Company concerning the Cost of Capital for Gas Local Distribution Company assets, November 2008 and May 2009.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER09-249-000, on behalf of Public Service Electric and Gas Company, on the incentive Cost of Capital for Mid-Atlantic Power Pathway Electric Transmission Assets, November 2008.

Direct and rebuttal testimony before the Public Utilities Commission of Ohio, Case No. 08-935-EL-SSO, on behalf of Ohio Edison Company, The Toledo Edison Company, and The Cleveland Electric Illuminating Company, with regard to the test to determine Significantly Excessive Earnings within the context of Senate Bill No. 221, September 2008 and October 2008.

Direct and rebuttal testimony before the Public Service Commission of West Virginia, Case No. 08-0900-W-42t, on behalf of West Virginia-American Water Company concerning the Cost of Capital for Water Utility assets, July 2008 and November 2008.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER08-1233-000, on behalf of Public Service Electric and Gas Company, on the Cost of Capital for Electric Transmission Assets, July 2008.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER08-1207-000, on behalf of Virginia Electric and Power Company, on the incentive Cost of Capital for investment in New Electric Transmission Assets, June 2008.

Direct and rebuttal testimony before the Federal Energy Regulatory Commission, Docket No. RP08-426-000, on behalf of El Paso Natural Gas Company, on the Cost of Capital for Natural Gas Transmission Assets, June 2008 and August 2009.

Rebuttal testimony on the financial risk of Purchased Power Agreements, before the Public Utilities Commission of the State of Colorado, Docket No. 07A-447E, in the matter of the application of Public Service Company of Colorado for approval of its 2007 Colorado Resource Plan, June 2008.

Direct and rebuttal testimony before the California Public Utilities Commission, Docket No. A.08-05-003, on behalf of California-American Water Company, concerning Cost of Capital, May 2008 and August 2008.

Post-Technical Conference Affidavit on behalf of The Interstate Natural Gas Association of America in response to the Reply Comments of the State of Alaska with regard the FERC=s Proposed Policy Statement on to the Composition of Proxy Companies for Determining Gas and Oil Pipeline Return on Equity, Docket No. PL07-2-000, March, 2008.

Direct and rebuttal testimony on the Cost of Capital before the Tennessee Regulatory Authority, Case No. 08-00039, on behalf of Tennessee American Water Company, March and August 2008.

Comments in support of The Interstate Natural Gas Association of America=s Additional Initial Comments on the FERC=s Proposed Policy Statement with regard to the Composition of Proxy Companies for Determining Gas and Oil Pipeline Return on Equity, Docket No. PL07-2-000, December, 2007.

Written direct and reply evidence before the National Energy Board in the matter of the National Energy Board Act, R.S.C. 1985, c. NB7, as amended, and the Regulations made thereunder; and in the matter of an application by Trans Québec & Maritimes PipeLines Inc. ("TQM") for orders pursuant to Part I and Part IV of the *National Energy Board Act*, for determining the overall fair return on capital for tolls charged by TQM, December 2007 and September 2008, Decision RH-1-2008, dated March 2009.

Direct and rebuttal testimony before the California Public Utilities Commission, Docket No. A. 07-01-022, on behalf of California-American Water Company, on the Effect of a Water Revenue Adjustment Mechanism on the Cost of Capital, October 2007 and November 2007.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER08-92-000 to Docket No. ER08-92-003, on behalf of Virginia Electric and Power Company, on the Cost of Capital for Transmission Assets, October 2007.

Direct and Supplemental testimony before the Public Utilities Commission of Ohio, Case No. 07-829-GA-AIR, Case No. 07-830-GA-ALT, and Case No. 07-831-GA-AAM, on behalf of Dominion East Ohio Company, on the rate of return for Dominion East Ohio=s natural gas distribution operations, September 2007 and June 2008.

Direct and rebuttal testimony before the State Corporation Commission of Virginia, Case No. PUE-2007-00066, on behalf of Virginia Electric and Power Company on the cost of capital for its southwest Virginia coal plant, July 2007 and December 2007.

Direct testimony before the Public Service Commission of West Virginia, Case No. 07-0998-W-42T, on behalf of West Virginia American Water Company on cost of capital, July 2007.

Direct, supplemental and rebuttal testimony before the Public Utilities Commission of Ohio, Case No. 07-551-EL-AIR, Case No. 07-552-EL-ATA, Case No. 07-553-EL-AAM, and Case No. 07-554-EL-UNC, on behalf of Ohio Edison Company, The Toledo Edison Company, and The Cleveland Electric Illuminating Company, on the cost of capital for the FirstEnergy Company=s Ohio electric distribution utilities, June 2007, January 2008 and February 2008.

Direct testimony before the Public Utilities Commission of the State of South Dakota, Docket No. NG-07-013, on behalf of NorthWestern Corporation, on the Cost of Capital for NorthWestern Energy Company=s natural gas operations in South Dakota, June 2007.

Rebuttal testimony before the California Public Utilities Commission, Docket No. A. 07-01-036-39, on behalf of California-American Water Company, on the Cost of Capital, May 2007.

Direct and rebuttal testimony before the Public Service Commission of Wisconsin, Docket No. 5-UR-103, on behalf of Wisconsin Energy Corporation, on the Cost of Capital for Wisconsin Electric Power Company and Wisconsin Gas LLC, May 2007 and October 2007.

Direct and rebuttal testimony before the Tennessee Regulatory Authority, Case No. 06-00290, on behalf of Tennessee American Water Company, on the Cost of Capital, November, 2006 and April 2007.

Direct testimony before the Federal Energy Regulatory Commission, Docket No. ER07-46-000, on behalf of Northwestern Corporation on the Cost of Capital for Transmission Assets, October 2006.

Direct and supplemental testimony before the Federal Energy Regulatory Commission, Docket No. ER06-427-003, on behalf of Mystic Development, LLC on the Cost of Capital for Mystic 8 and 9 Generating Plants Operating Under Reliability Must Run Contract, August 2006 and September 2006.

Expert report in the United States Tax Court, Docket No. 21309-05, 34th Street Partners, DH Petersburg Investment, LLC and Mid-Atlantic Finance, Partners Other than the Tax Matters Partner, Petitioner, v. Commissioner of Internal Revenue, Respondent, July 28, 2006.

Direct and rebuttal testimony before the Pennsylvania Public Utility Commission, Return on Equity for Metropolitan Edison Company, Docket No. R-00061366 and Pennsylvania Electric Company, Docket No. R-00061367, April 2006 and August 2006.

Written evidence before the Ontario Energy Board, Cost of Capital for Union Gas Limited, Inc., Docket No. EB-2005-0520, January 2006.

Direct testimony before the Arizona Corporation Commission, Cost of Capital for Paradise Valley Water Company, a subsidiary of Arizona-American Water Company, Docket No. WS-01303A-05, May 2005.

Direct and rebuttal testimony before the Federal Energy Regulatory Commission on Energy Allocation of Debt Cost for Incremental Shipping Rates for Edison Mission Energy, Docket No. RP04-274-000, December 2004 and March 2005.

Direct and rebuttal testimony before the Public Service Commission of West Virginia, on Cost of Capital for West Virginia-American Water Company, Case No 04-0373-W-42T, May 2004.

Written evidence before the National Energy Board in the matter of the National Energy Board Act, R.S.C. 1985, c. NB7, as amended, (Act) and the Regulations made under it; and in the matter of an application by TransCanada PipeLines Limited for orders pursuant to Part IV of the *National Energy Board Act*, for approval of Mainline Tolls for 2004, RH-2-2004, January 2004.

Direct and rebuttal reports before the Alberta Energy and Utilities Board in the matter of the Alberta Energy and Utilities Board Act, R.S.A. 2000, c. A-17, and the Regulations under it; in the matter of the Gas Utilities Act, R.S.A. 2000, c. G-5, and the Regulations under it; in the matter of the Public Utilities Board Act, R.S.A. 2000, c. P-45, as amended, and the Regulations under it; and in the matter of Alberta Energy and Utilities Generic Cost of Capital Hearing, Application No. 1271597, July 2003, November 2003, Decision 2004-052, dated July 2004.

Direct report before the Arbitration Panel in the arbitration of stranded costs for the Town of Belleair, FL, Case No. 000-6487-C1-007, April 2003.

Direct testimony before the Federal Energy Regulatory Commission on behalf of Florida Power Corporation, dba Progress Energy Florida, Inc. in Docket No. SC03-1-000, March 2003.

Direct testimony and hearing before the Arbitration Panel in the arbitration of stranded costs for the City of Winter Park, FL, In the Circuit Court of the Ninth Judicial Circuit in and for Orange County, FL, Case No. C1-01-4558-39, December 2002.

Direct reports before the Arbitration Board for Petroleum products trade in the Arbitration of the Military Sealift Command vs. Household Commercial Financial Services, fair value of sale of the Darnell, October 2002.

Direct and rebuttal reports before the Arbitration Panel in the arbitration of stranded costs for the City of Casselberry, FL, Case No. 00-CA-1107-16-L, July 2002.

Direct testimony (with William Lindsay) before the Federal Energy Regulatory Commission on behalf of DTE East China, LLC in Docket No. ER02-1599-000, April 2002.

Written evidence before the Public Utility Board on behalf of Newfoundland & Labrador Hydro - Rate Hearings, October 2001, Order No. P.U.7 (2002-2003), dated June 2002.

Written evidence, rebuttal, reply and further reply before the National Energy Board in the matter of an application by TransCanada PipeLines Limited for orders pursuant to Part I and Part IV of the *National Energy Board Act*, Order AO-1-RH-4-2001, May 2001, Nov. 2001, Feb. 2002.

Direct testimony before the Federal Energy Regulatory Commission on behalf of Mississippi River Transmission Corporation in Docket No. RP01-292-000, March 2001.

Direct testimony before the Alberta Energy and Utilities Board on behalf of TransAlta Utilities Corporation for approval of its 2001 transmission tariff, May 2000.

Direct testimony before the Federal Energy Regulatory Commission on behalf of Central Maine Power in Docket No. ER00-982-000, December 1999.

Direct and rebuttal testimony before the Alberta Energy and Utilities Board on behalf of TransAlta Utilities Corporation in the matter of an application for approval of its 1999 and 2000 generation tariff, transmission tariff, and distribution revenue requirement, Docket U99099, October 1998.

Schedule No. D5

10.75% Cost of Common Shareholders' Equity

Schedule No. D5.1

Table of Contents

Schedule No. D5.2

Classification of Companies by Assets

Company	Company Category
Atmos Energy	R
Chesapeake Utilities	M
ONE Gas Inc.	R
South Jersey Inds.	M
Southwest Gas	R
Spire Inc.	R
New Jersey Resources	M
Northwest Natural Gas	R
WGL Holdings Inc.	R

Sources and Notes:
Percent regulated determined based on respective company
2016 10-K information.
R = Regulated (greater than 80 percent of total assets are regulated).
M = Mostly Regulated (50 to 80 percent of total assets are regulated).
D = Diversified (less than 50 percent of total assets are regulated).

Schedule No. D5.3

Market Value of the Expanded Sample

Panel A: Atmos Energy

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$3,899	$3,899	$3,699	$3,272	$3,064	$2,661	$2,424	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	106	106	105	102	101	91	91	[b]
Price per Share - Common	15_day_Average	$82	$88	$74	$63	$55	$45	$36	[c]
Market Value of Common Equity		$8,649	$9,303	$7,778	$6,398	$5,523	$4,061	$3,223	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$8,649	$9,303	$7,778	$6,398	$5,523	$4,061	$3,223	[f]= [d]
Market to Book Value of Common Equity		2.22	2.39	2.10	1.96	1.80	1.53	1.33	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$540	$540	$979	$863	$1,119	$1,300	$1,165	[j]
Current Liabilities	BS_CUR_LIAB	$1,013	$1,013	$1,950	$1,515	$1,421	$2,014	$1,645	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$0	$0	$250	$0	$0	$500	$0	[l]
Net Working Capital		($474)	($474)	($720)	($652)	($302)	($214)	($480)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$448	$448	$941	$763	$551	$690	$831	[n]
Adjusted Short-Term Debt		$448	$448	$720	$652	$302	$214	$480	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$3,067	$3,067	$2,314	$2,455	$2,455	$1,956	$1,956	[p]
Book Value of Long-Term Debt		$3,515	$3,515	$3,285	$3,107	$2,757	$2,670	$2,437	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$2,845	$2,845	$2,669	$2,770	$2,676	$2,426	$2,561	
Carrying Amount		$2,460	$2,460	$2,460	$2,460	$2,460	$1,960	$2,213	
Adjustment to Book Value of Long-Term Debt		$385	$385	$209	$310	$216	$466	$348	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$3,900	$3,900	$3,494	$3,417	$2,973	$3,136	$2,785	[s] = [q] + [r].
Market Value of Debt		$3,900	$3,900	$3,494	$3,417	$2,973	$3,136	$2,785	[t] = [s].
MARKET VALUE OF FIRM		$12,549	$13,203	$11,272	$9,815	$8,496	$7,197	$6,008	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		68.92%	70.46%	69.00%	65.19%	65.00%	56.43%	53.65%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		31.08%	29.54%	31.00%	34.81%	35.00%	43.57%	46.35%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel B: Chesapeake Utilities

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$464	$464	$446	$358	$300	$279	$257	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	16	16	16	15	15	14	14	[b]
Price per Share - Common	15_day_Average	$73	$79	$68	$56	$49	$39	$30	[c]
Market Value of Common Equity		$1,192	$1,293	$1,104	$851	$710	$570	$435	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$1,192	$1,293	$1,104	$851	$710	$570	$435	[f]= [d]
Market to Book Value of Common Equity		2.57	2.79	2.48	2.38	2.36	2.04	1.70	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$149	$149	$141	$112	$122	$126	$101	[j]
Current Liabilities	BS_CUR_LIAB	$339	$339	$334	$280	$194	$222	$162	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$12	$12	$12	$9	$9	$11	$8	[l]
Net Working Capital		($178)	($178)	($181)	($159)	($63)	($84)	($53)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$203	$203	$210	$173	$88	$106	$61	[n]
Adjusted Short-Term Debt		$178	$178	$181	$159	$63	$84	$53	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$201	$201	$137	$149	$158	$118	$102	[p]
Book Value of Long-Term Debt		$392	$392	$330	$317	$230	$213	$163	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$162	$162	$165	$181	$137	$133	$142	
Carrying Amount		$146	$146	$154	$162	$122	$110	$119	
Adjustment to Book Value of Long-Term Debt		$16	$16	$11	$19	$15	$23	$24	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$407	$407	$341	$336	$245	$236	$187	[s] = [q] + [r].
Market Value of Debt		$407	$407	$341	$336	$245	$236	$187	[t] = [s].
MARKET VALUE OF FIRM		$1,599	$1,700	$1,446	$1,188	$955	$806	$622	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		74.53%	76.06%	76.39%	71.70%	74.34%	70.69%	69.91%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		25.47%	23.94%	23.61%	28.30%	25.66%	29.31%	30.09%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel C: ONE Gas Inc.

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$1,932	$1,932	$1,888	$1,842	$1,794	n/a	n/a	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	52	52	52	52	52	n/a	n/a	[b]
Price per Share - Common	15_day_Average	$70	$75	$64	$49	$42	n/a	n/a	[c]
Market Value of Common Equity		$3,644	$3,901	$3,324	$2,577	$2,188	n/a	n/a	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$3,644	$3,901	$3,324	$2,577	$2,188	n/a	n/a	[f]= [d]
Market to Book Value of Common Equity		1.89	2.02	1.76	1.40	1.22	n/a	n/a	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	n/a	n/a	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	n/a	n/a	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$446	$446	$569	$483	$668	n/a	n/a	[j]
Current Liabilities	BS_CUR_LIAB	$392	$392	$444	$304	$392	n/a	n/a	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$0	$0	$0	$0	$0	n/a	n/a	[l]
Net Working Capital		$53	$53	$125	$179	$275	n/a	n/a	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$174	$174	$145	$13	$42	n/a	n/a	[n]
Adjusted Short-Term Debt		$0	$0	$0	$0	$0	n/a	n/a	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$1,193	$1,193	$1,192	$1,192	$1,201	n/a	n/a	[p]
Book Value of Long-Term Debt		$1,193	$1,193	$1,192	$1,192	$1,201	n/a	n/a	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$1,200	$1,200	$1,200	$1,300	$1,200	n/a	n/a	
Carrying Amount		$1,200	$1,200	$1,200	$1,200	$1,000	n/a	n/a	
Adjustment to Book Value of Long-Term Debt		$0	$0	$0	$100	$200	n/a	n/a	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$1,193	$1,193	$1,192	$1,292	$1,401	n/a	n/a	[s] = [q] + [r].
Market Value of Debt		$1,193	$1,193	$1,192	$1,292	$1,401	n/a	n/a	[t] = [s].
MARKET VALUE OF FIRM									
		$4,837	$5,094	$4,517	$3,868	$3,590	n/a	n/a	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		75.34%	76.58%	73.60%	66.61%	60.96%	n/a	n/a	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	n/a	n/a	[w] = [i] / [u].
Debt - Market Value Ratio		24.66%	23.42%	26.40%	33.39%	39.04%	n/a	n/a	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel D: South Jersey Inds.

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$1,221	$1,221	$1,289	$1,038	$932	$827	$736	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	80	80	79	71	68	65	63	[b]
Price per Share - Common	15_day_Average	$30	$32	$34	$23	$29	$28	$25	[c]
Market Value of Common Equity		$2,350	$2,516	$2,719	$1,648	$1,999	$1,805	$1,600	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$2,350	$2,516	$2,719	$1,648	$1,999	$1,805	$1,600	[f]= [d]
Market to Book Value of Common Equity		1.92	2.06	2.11	1.59	2.14	2.18	2.17	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$323	$323	$473	$431	$567	$483	$395	[j]
Current Liabilities	BS_CUR_LIAB	$684	$684	$953	$832	$850	$765	$652	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$11	$11	$232	$29	$150	$21	$25	[l]
Net Working Capital		($350)	($350)	($247)	($372)	($134)	($261)	($232)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$280	$280	$296	$432	$246	$354	$339	[n]
Adjusted Short-Term Debt		$280	$280	$247	$372	$134	$261	$232	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$1,180	$1,180	$808	$997	$879	$701	$601	[p]
Book Value of Long-Term Debt		$1,471	$1,471	$1,287	$1,399	$1,163	$983	$858	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$1,081	$1,081	$1,079	$1,059	$713	$682	$533	
Carrying Amount		$1,047	$1,047	$1,036	$1,009	$701	$626	$426	
Adjustment to Book Value of Long-Term Debt		$33	$33	$43	$49	$12	$56	$107	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$1,505	$1,505	$1,331	$1,448	$1,174	$1,039	$965	[s] = [q] + [r].
Market Value of Debt		$1,505	$1,505	$1,331	$1,448	$1,174	$1,039	$965	[t] = [s].
MARKET VALUE OF FIRM									
		$3,855	$4,021	$4,049	$3,096	$3,173	$2,844	$2,565	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		60.97%	62.58%	67.14%	53.23%	62.99%	63.47%	62.37%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		39.03%	37.42%	32.86%	46.77%	37.01%	36.53%	37.63%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel E: Southwest Gas

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$1,716	$1,716	$1,663	$1,594	$1,489	$1,415	$1,310	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	48	48	47	47	47	46	46	[b]
Price per Share - Common	15_day_Average	$75	$81	$76	$53	$60	$54	$42	[c]
Market Value of Common Equity		$3,581	$3,860	$3,606	$2,528	$2,771	$2,506	$1,952	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$3,581	$3,860	$3,606	$2,528	$2,771	$2,506	$1,952	[f]= [d]
Market to Book Value of Common Equity		2.09	2.25	2.17	1.59	1.86	1.77	1.49	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$539	$539	$533	$558	$607	$495	$458	[j]
Current Liabilities	BS_CUR_LIAB	$656	$656	$628	$535	$470	$434	$535	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$28	$28	$50	$19	$19	$11	$50	[l]
Net Working Capital		($89)	($89)	($45)	$43	$156	$72	($27)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$111	$111	$0	$18	$5	$0	$0	[n]
Adjusted Short-Term Debt		$89	$89	$0	$0	$0	$0	$0	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$1,732	$1,732	$1,550	$1,551	$1,631	$1,381	$1,268	[p]
Book Value of Long-Term Debt		$1,849	$1,849	$1,600	$1,571	$1,651	$1,392	$1,319	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$1,680	$1,680	$1,646	$1,796	$1,463	$1,482	$1,319	
Carrying Amount		$1,550	$1,550	$1,551	$1,657	$1,392	$1,319	$1,253	
Adjustment to Book Value of Long-Term Debt		$130	$130	$94	$139	$71	$164	$66	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$1,979	$1,979	$1,695	$1,710	$1,722	$1,556	$1,384	[s] = [q] + [r].
Market Value of Debt		$1,979	$1,979	$1,695	$1,710	$1,722	$1,556	$1,384	[t] = [s].
MARKET VALUE OF FIRM		$5,560	$5,839	$5,300	$4,238	$4,492	$4,062	$3,336	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		64.40%	66.10%	68.03%	59.65%	61.68%	61.70%	58.50%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		35.60%	33.90%	31.97%	40.35%	38.32%	38.30%	41.50%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel F: Spire Inc.

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$2,079	$2,079	$1,797	$1,600	$1,534	$1,066	$621	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	48	48	46	43	43	33	23	[b]
Price per Share - Common	15_day_Average	$69	$76	$64	$58	$52	$45	$39	[c]
Market Value of Common Equity		$3,322	$3,677	$2,935	$2,533	$2,261	$1,482	$877	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$3,322	$3,677	$2,935	$2,533	$2,261	$1,482	$877	[f]= [d]
Market to Book Value of Common Equity		1.60	1.77	1.63	1.58	1.47	1.39	1.41	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$853	$853	$816	$636	$816	$571	$400	[j]
Current Liabilities	BS_CUR_LIAB	$1,211	$1,211	$1,342	$848	$1,082	$478	$275	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$106	$106	$250	$0	$115	$80	$25	[l]
Net Working Capital		($253)	($253)	($277)	($212)	($152)	$173	$150	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$584	$584	$506	$377	$398	$94	$83	[n]
Adjusted Short-Term Debt		$253	$253	$277	$212	$152	$0	$0	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$2,030	$2,030	$1,821	$1,852	$1,736	$833	$364	[p]
Book Value of Long-Term Debt		$2,389	$2,389	$2,348	$2,063	$2,003	$913	$389	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$2,257	$2,257	$1,944	$1,937	$954	$453	$444	
Carrying Amount		$2,084	$2,084	$1,852	$1,851	$913	$364	$364	
Adjustment to Book Value of Long-Term Debt		$173	$173	$93	$86	$41	$88	$79	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$2,562	$2,562	$2,441	$2,149	$2,044	$1,001	$469	[s] = [q] + [r].
Market Value of Debt		$2,562	$2,562	$2,441	$2,149	$2,044	$1,001	$469	[t] = [s].
MARKET VALUE OF FIRM		$5,884	$6,239	$5,376	$4,682	$4,305	$2,483	$1,346	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		56.45%	58.93%	54.60%	54.09%	52.53%	59.68%	65.18%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		43.55%	41.07%	45.40%	45.91%	47.47%	40.32%	34.82%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel G: New Jersey Resources

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$1,237	$1,237	$1,185	$1,144	$1,104	$882	$863	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	87	87	86	86	85	84	84	[b]
Price per Share - Common	15_day_Average	$39	$40	$36	$31	$30	$22	$20	[c]
Market Value of Common Equity		$3,401	$3,499	$3,119	$2,663	$2,554	$1,892	$1,678	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$3,401	$3,499	$3,119	$2,663	$2,554	$1,892	$1,678	[f]= [d]
Market to Book Value of Common Equity		2.75	2.83	2.63	2.33	2.31	2.15	1.94	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$579	$579	$815	$589	$900	$898	$771	[j]
Current Liabilities	BS_CUR_LIAB	$803	$803	$823	$575	$816	$1,053	$797	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$165	$165	$97	$11	$35	$70	$9	[l]
Net Working Capital		($58)	($58)	$89	$25	$120	($85)	($17)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$266	$266	$285	$211	$254	$504	$394	[n]
Adjusted Short-Term Debt		$58	$58	$0	$0	$0	$85	$17	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$997	$997	$1,027	$848	$703	$518	$530	[p]
Book Value of Long-Term Debt		$1,221	$1,221	$1,124	$859	$738	$672	$555	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$732	$732	$584	$587	$557	$530	$416	
Carrying Amount		$708	$708	$583	$558	$530	$480	$380	
Adjustment to Book Value of Long-Term Debt		$24	$24	$1	$29	$27	$50	$37	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$1,244	$1,244	$1,125	$888	$765	$722	$592	[s] = [q] + [r].
Market Value of Debt		$1,244	$1,244	$1,125	$888	$765	$722	$592	[t] = [s].
MARKET VALUE OF FIRM		$4,645	$4,743	$4,244	$3,551	$3,319	$2,615	$2,270	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		73.21%	73.77%	73.49%	74.99%	76.95%	72.37%	73.91%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		26.79%	26.23%	26.51%	25.01%	23.05%	27.63%	26.09%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel H: Northwest Natural Gas

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$847	$847	$850	$781	$767	$752	$730	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	29	29	29	27	27	27	27	[b]
Price per Share - Common	15_day_Average	$57	$62	$60	$50	$49	$43	$44	[c]
Market Value of Common Equity		$1,639	$1,776	$1,726	$1,369	$1,340	$1,155	$1,191	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$1,639	$1,776	$1,726	$1,369	$1,340	$1,155	$1,191	[f]= [d]
Market to Book Value of Common Equity		1.94	2.10	2.03	1.75	1.75	1.54	1.63	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$0	$0	$0	$0	$0	$0	$0	[h]
Market Value of Preferred Equity		$0	$0	$0	$0	$0	$0	$0	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$200	$200	$288	$331	$363	$330	$284	[j]
Current Liabilities	BS_CUR_LIAB	$203	$203	$275	$478	$469	$433	$368	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$22	$22	$40	$25	$40	$60	$0	[l]
Net Working Capital		$19	$19	$54	($122)	($67)	($42)	($85)	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$0	$0	$53	$270	$235	$188	$190	[n]
Adjusted Short-Term Debt		$0	$0	$0	$122	$67	$42	$85	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$757	$757	$679	$569	$622	$682	$692	[p]
Book Value of Long-Term Debt		$779	$779	$719	$716	$729	$784	$776	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$793	$793	$667	$757	$806	$835	$809	
Carrying Amount		$719	$719	$602	$662	$742	$692	$682	
Adjustment to Book Value of Long-Term Debt		$74	$74	$65	$95	$65	$143	$127	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$853	$853	$785	$811	$793	$927	$903	[s] = [q] + [r].
Market Value of Debt		$853	$853	$785	$811	$793	$927	$903	[t] = [s].
MARKET VALUE OF FIRM		$2,492	$2,630	$2,511	$2,181	$2,133	$2,082	$2,095	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		65.76%	67.54%	68.74%	62.79%	62.81%	55.48%	56.87%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		-	-	-	-	-	-	-	[w] = [i] / [u].
Debt - Market Value Ratio		34.24%	32.46%	31.26%	37.21%	37.19%	44.52%	43.13%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.3

Market Value of the Expanded Sample

Panel I: WGL Holdings Inc.

($MM)

		DCF Capital Structure	Year End, 2017	Year End, 2016	Year End, 2015	Year End, 2014	Year End, 2013	Year End, 2012	Notes
MARKET VALUE OF COMMON EQUITY									
Book Value, Common Shareholder's Equity	TOT_COMMON_EQY	$1,503	$1,503	$1,439	$1,289	$1,243	$1,273	$1,303	[a]
Shares Outstanding (in millions) - Common	BS_SH_OUT	51	51	51	50	50	52	52	[b]
Price per Share - Common	15_day_Average	$85	$86	$78	$63	$54	$39	$39	[c]
Market Value of Common Equity		$4,340	$4,392	$3,985	$3,126	$2,660	$2,025	$2,032	[d] = [b] x [c].
Market Value of GP Equity		n/a	n/a	n/a	n/a	n/a	n/a	n/a	[e]
Total Market Value of Equity		$4,340	$4,392	$3,985	$3,126	$2,660	$2,025	$2,032	[f]= [d]
Market to Book Value of Common Equity		2.89	2.92	2.77	2.43	2.14	1.59	1.56	[g] = [f] / [a].
MARKET VALUE OF PREFERRED EQUITY									
Book Value of Preferred Equity	BS_PFD_EQY	$28	$28	$28	$28	$28	$28	$28	[h]
Market Value of Preferred Equity		$28	$28	$28	$28	$28	$28	$28	[i] = [h].
MARKET VALUE OF DEBT									
Current Assets	BS_CUR_ASSET_REPORT	$986	$986	$1,095	$918	$1,028	$1,040	$1,011	[j]
Current Liabilities	BS_CUR_LIAB	$1,489	$1,489	$1,422	$1,181	$1,033	$1,090	$970	[k]
Current Portion of Long-Term Debt	BS_ST_PORTION_OF_LT_DEBT	$250	$250	$0	$25	$20	$30	$37	[l]
Net Working Capital		($253)	($253)	($327)	($238)	$15	($20)	$78	[m] = [j] - ([k] - [l]).
Notes Payable (Short-Term Debt)	BS_ST_DEBT	$560	$560	$634	$528	$350	$443	$361	[n]
Adjusted Short-Term Debt		$253	$253	$327	$238	$0	$20	$0	[o] = See Sources and Notes.
Long-Term Debt	BS_LT_BORROW	$1,431	$1,431	$1,435	$946	$976	$599	$554	[p]
Book Value of Long-Term Debt		$1,934	$1,934	$1,762	$1,209	$996	$649	$591	[q] = [l] + [o] + [p].
Unadjusted Market Value of Long Term Debt		$1,642	$1,642	$1,058	$809	$630	$759	$721	
Carrying Amount		$1,444	$1,444	$944	$679	$524	$589	$587	
Adjustment to Book Value of Long-Term Debt		$198	$198	$114	$130	$106	$170	$134	[r] = See Sources and Notes.
Market Value of Long-Term Debt		$2,132	$2,132	$1,876	$1,339	$1,102	$819	$724	[s] = [q] + [r].
Market Value of Debt		$2,132	$2,132	$1,876	$1,339	$1,102	$819	$724	[t] = [s].
MARKET VALUE OF FIRM									
		$6,500	$6,552	$5,889	$4,493	$3,790	$2,872	$2,785	[u] = [f] + [i] + [t].
DEBT AND EQUITY TO MARKET VALUE RATIOS									
Common Equity - Market Value Ratio		66.77%	67.04%	67.67%	69.58%	70.18%	70.50%	72.98%	[v] = [f] / [u].
Preferred Equity - Market Value Ratio		0.43%	0.43%	0.48%	0.63%	0.74%	0.98%	1.01%	[w] = [i] / [u].
Debt - Market Value Ratio		32.80%	32.53%	31.85%	29.79%	29.07%	28.51%	26.01%	[x] = [t] / [u].

Sources and Notes:
Bloomberg as of January 31, 2018
Capital structure from Year End, 2017 calculated using respective balance sheet information and 15-day average prices ending at period end.
The DCF Capital structure is calculated using 4th Quarter, 2017 balance sheet information and a 15-trading day average closing price ending on 1/31/2018.
Prices are reported in Supporting Schedule #1 to Schedule No. D5.6.
[o] =
(1): 0 if [m] > 0.
(2): The absolute value of [m] if [m] < 0 and |[m]| < [n].
(3): [n] if [m] < 0 and |[m]| > [n].
[r]: Difference between fair value of Long-Term debt and carrying amount of Long-Term debt per company 10-K. Data for adjustment is from 2017 10-K.

Schedule No. D5.4

Capital Structure Summary of the Expanded Sample

Company	DCF Capital Structure			5-Year Average Capital Structure		
	Common Equity - Value Ratio [1]	Preferred Equity - Value Ratio [2]	Debt - Value Ratio [3]	Common Equity - Value Ratio [4]	Preferred Equity - Value Ratio [5]	Debt - Value Ratio [6]
Atmos Energy	68.9%	0.0%	31.1%	63.5%	0.0%	36.5%
Chesapeake Utilities	74.5%	0.0%	25.5%	73.2%	0.0%	26.8%
ONE Gas Inc.	75.3%	0.0%	24.7%	47.9%	0.0%	22.1%
South Jersey Inds.	61.0%	0.0%	39.0%	61.9%	0.0%	38.1%
Southwest Gas	64.4%	0.0%	35.6%	62.7%	0.0%	37.3%
Spire Inc.	56.5%	0.0%	43.5%	56.6%	0.0%	43.4%
New Jersey Resources	73.2%	0.0%	26.8%	74.3%	0.0%	25.7%
Northwest Natural Gas	65.8%	0.0%	34.2%	62.4%	0.0%	37.6%
WGL Holdings Inc.	66.8%	0.4%	32.8%	69.6%	0.7%	29.7%
Average	67.4%	0.0%	32.6%	63.6%	0.1%	33.0%
Subsample Average	67.6%	0.0%	32.4%	61.1%	0.0%	33.9%

Sources and Notes:
[1], [4]: Supporting Schedule #1 to Schedule No. D5.4.
[2], [5]: Supporting Schedule #2 to Schedule No. D5.4.
[3], [6]: Supporting Schedule #3 to Schedule No. D5.4.
Values in this table may not add up exactly to 100% because of rounding.

Schedule No. D5.5

Estimated Growth Rates of the Expanded Sample

Company	ThomsonOne IBES Estimate		Value Line			
	Long-Term Growth Rate	Number of Estimates	EPS Year 2017 Estimate	EPS Year 2020-2022 Estimate	Annualized Growth Rate	Combined Growth Rate
	[1]	[2]	[3]	[4]	[5]	[6]
Atmos Energy	6.5%	1	$3.80	$4.50	4.3%	5.4%
Chesapeake Utilities	n/a	n/a	$2.65	$4.20	12.2%	12.2%
ONE Gas Inc.	6.0%	1	$2.95	$4.00	7.9%	7.0%
South Jersey Inds.	n/a	n/a	$1.15	$2.00	14.8%	14.8%
Southwest Gas	n/a	n/a	$3.55	$4.80	7.8%	7.8%
Spire Inc.	4.5%	2	$3.80	$4.65	5.2%	4.7%
New Jersey Resources	n/a	n/a	$1.90	$2.05	1.9%	1.9%
Northwest Natural Gas	n/a	n/a	$2.25	$3.15	8.8%	8.8%
WGL Holdings Inc.	n/a	n/a	$3.50	$3.45	-0.4%	-0.4%

Sources and Notes:

[1] - [2]: Updated from ThomsonOne as of Jan 31, 2018.

[3] - [4]: From Valueline Investment Analyzer as of Jan 8, 2018.

[5]: ([4]/[3])^(1/4) - 1, where 4 is the number of years between 2021, the middle year of Value Line's 3-5 year forecast, and our study year 2017.

[6]: Weighted average growth rate.

Schedule No. D5.6

DCF Cost of Equity of the Expanded Sample

Panel A: Simple DCF Method (Quarterly)

Company	Stock Price [1]	Most Recent Dividend [2]	Quarterly Dividend Yield (t+1) [3]	Combined Long-Term Growth Rate [4]	Quarterly Growth Rate [5]	DCF Cost of Equity [6]
Atmos Energy	$81.51	$0.49	0.60%	5.4%	1.3%	7.9%
Chesapeake Utilities	$72.90	$0.33	0.46%	12.2%	2.9%	14.2%
ONE Gas Inc.	$69.72	$0.42	0.61%	7.0%	1.7%	9.6%
South Jersey Inds.	$29.55	$0.28	0.98%	14.8%	3.5%	19.3%
Southwest Gas	$75.01	$0.50	0.67%	7.8%	1.9%	10.7%
Spire Inc.	$68.72	$0.56	0.83%	4.7%	1.2%	8.2%
New Jersey Resources	$39.29	$0.27	0.70%	1.9%	0.5%	4.8%
Northwest Natural Gas	$57.07	$0.47	0.85%	8.8%	2.1%	12.4%
WGL Holdings Inc.	$84.73	$0.51	0.60%	-0.4%	-0.1%	2.1%

Sources and Notes:
[1]: Supporting Schedule #1 to Schedule No. D5.6.
[2]: Supporting Schedule #2 to Schedule No. D5.6.
[3]: ([2] / [1]) x (1 + [5]).
[4]: Schedule No. D5.5, [6].
[5]: {(1 + [4]) ^ (1/4)} - 1.
[6]: {([3] + [5] + 1) ^ 4} - 1.

Schedule No. D5.6

DCF Cost of Equity of the Expanded Sample

Panel B: Multi-Stage DCF (Using Blue Chip Economic Indicators, October 2017 U.S. GDP Growth Forecast as the Perpetual Rate)

Company	Stock Price [1]	Most Recent Dividend [2]	Combined Long-Term Growth Rate [3]	Growth Rate: Year 6 [4]	Growth Rate: Year 7 [5]	Growth Rate: Year 8 [6]	Growth Rate: Year 9 [7]	Growth Rate: Year 10 [8]	GDP Long-Term Growth Rate [9]	DCF Cost of Equity [10]
Atmos Energy	$81.51	$0.49	5.41%	5.21%	5.01%	4.80%	4.60%	4.40%	4.20%	6.9%
Chesapeake Utilities	$72.90	$0.33	12.20%	10.87%	9.53%	8.20%	6.87%	5.53%	4.20%	7.3%
ONE Gas Inc.	$69.72	$0.42	6.95%	6.50%	6.04%	5.58%	5.12%	4.66%	4.20%	7.2%
South Jersey Inds.	$29.55	$0.28	14.84%	13.06%	11.29%	9.52%	7.75%	5.97%	4.20%	11.6%
Southwest Gas	$75.01	$0.50	7.83%	7.23%	6.62%	6.02%	5.41%	4.81%	4.20%	7.7%
Spire Inc.	$68.72	$0.56	4.71%	4.63%	4.54%	4.46%	4.37%	4.29%	4.20%	7.8%
New Jersey Resources	$39.29	$0.27	1.92%	2.30%	2.68%	3.06%	3.44%	3.82%	4.20%	6.7%
Northwest Natural Gas	$57.07	$0.47	8.78%	8.01%	7.25%	6.49%	5.73%	4.96%	4.20%	8.8%
WGL Holdings Inc.	$84.73	$0.51	-0.36%	0.40%	1.16%	1.92%	2.68%	3.44%	4.20%	6.1%

Sources and Notes:
[1]: Supporting Schedule #1 to Schedule No. D5.6.
[2]: Supporting Schedule #2 to Schedule No. D5.6.
[3]: Schedule No. D5.5, [6].
[4]: [3] - {([3] - [9])/ 6}.
[5]: [4] - {([3] - [9])/ 6}.
[6]: [5] - {([3] - [9])/ 6}.
[7]: [6] - {([3] - [9])/ 6}.
[8]: [7] - {([3] - [9])/ 6}.
[9]: Blue Chip Economic Indicators, October 2017 U.S. This number is assumed to be the perpetual growth rate.
[10]: Supporting Schedule #3 to Schedule No. D5.6.

Schedule No. D5.7

Overall After-Tax DCF Cost of Capital of the Expanded Sample

Panel A: Simple DCF Method (Quarterly)

Company	4th Quarter, 2017 Bond Rating [1]	4th Quarter, 2017 Preferred Equity Rating [2]	DCF Cost of Equity [3]	DCF Common Equity to Market Value Ratio [4]	Cost of Preferred Equity [5]	DCF Preferred Equity to Market Value Ratio [6]	DCF Cost of Debt [7]	DCF Debt to Market Value Ratio [8]	VVC Representative Income Tax Rate [9]	Overall After-Tax Cost of Capital [10]
Atmos Energy	A	-	7.9%	68.9%	-	0.0%	3.9%	31.1%	21.0%	6.43%
Chesapeake Utilities	na	-	14.2%	74.5%	-	0.0%	NA	25.5%	21.0%	NA
ONE Gas Inc.	A	-	9.6%	75.3%	-	0.0%	3.9%	24.7%	21.0%	7.96%
South Jersey Inds.	BBB	-	19.3%	61.0%	-	0.0%	4.2%	39.0%	21.0%	13.03%
Southwest Gas	BBB	-	10.7%	64.4%	-	0.0%	4.2%	35.6%	21.0%	8.07%
Spire Inc.	A	-	8.2%	56.5%	-	0.0%	3.9%	43.5%	21.0%	5.96%
New Jersey Resources	na	-	4.8%	73.2%	-	0.0%	NA	26.8%	21.0%	NA
Northwest Natural Gas	A	-	12.4%	65.8%	-	0.0%	3.9%	34.2%	21.0%	9.22%
WGL Holdings Inc.	A	A	2.1%	66.8%	3.9%	0.4%	3.9%	32.8%	21.0%	~~2.40%~~
Simple Full Sample Average			11.3%	65.3%	NA	0.0%	4.0%	34.7%	21.0%	8.44%
Simple Subsample Average			9.8%	66.2%	NA	0.0%	3.9%	33.8%	21.0%	7.53%

Sources and Notes:
[1]: S&P Credit Ratings from Research Insight.
[2]: Preferred ratings were assumed equal to debt ratings.
[3]: Schedule No. D5.6; Panel A, [6].
[4]: Schedule No. D5.4, [1].
[5]: Supporting Schedule #2 to Schedule No. D5.11, Panel C.
[6]: Schedule No. D5.4, [2].
[7]: Supporting Schedule #2 to Schedule No. D5.11, Panel B.
[8]: Schedule No. D5.4, [3].
[9]: VVC Effective Corporate Tax Rate.
[10]: ([3] x [4]) + ([5] x [6]) + {[7] x [8] x (1 - [9])}. A strikethrough indicates the utility was excluded from the full sample average calculation as a result of its cost of equity not exceeding its cost of debt by 100 basis points.

Schedule No. D5.7

Overall After-Tax DCF Cost of Capital of the Expanded Sample

Panel B: Multi-Stage DCF (Using Blue Chip Economic Indicators, October 2017 U.S. GDP Growth Forecast as the Perpetual Rate)

Company	4th Quarter, 2017 Bond Rating [1]	4th Quarter, 2017 Preferred Equity Rating	DCF Cost of Equity [3]	DCF Common Equity to Market Value Ratio [4]	Cost of Preferred Equity [5]	DCF Preferred Equity to Market Value Ratio [6]	DCF Cost of Debt [7]	DCF Debt to Market Value Ratio [8]	VVC Representative Income Tax Rate [9]	Overall After-Tax Cost of Capital [10]
Atmos Energy	A	-	6.9%	68.9%	-	0.0%	3.9%	31.1%	21.0%	5.72%
Chesapeake Utilities	na	-	7.3%	74.5%	-	0.0%	NA	25.5%	21.0%	NA
ONE Gas Inc.	A	-	7.2%	75.3%	-	0.0%	3.9%	24.7%	21.0%	6.20%
South Jersey Inds.	BBB	-	11.6%	61.0%	-	0.0%	4.2%	39.0%	21.0%	8.34%
Southwest Gas	BBB	-	7.7%	64.4%	-	0.0%	4.2%	35.6%	21.0%	6.14%
Spire Inc.	A	-	7.8%	56.5%	-	0.0%	3.9%	43.5%	21.0%	5.72%
New Jersey Resources	na	-	6.7%	73.2%	-	0.0%	NA	26.8%	21.0%	NA
Northwest Natural Gas	A	-	8.8%	65.8%	-	0.0%	3.9%	34.2%	21.0%	6.86%
WGL Holdings Inc.	A	A	6.1%	66.8%	3.9%	0.4%	3.9%	32.8%	21.0%	5.06%
Multi Full Sample Average			8.0%	65.5%	3.9%	0.1%	4.0%	34.4%	21.0%	6.3%
Multi Subsample Average			7.7%	66.2%	NA	0.00%	3.9%	33.8%	21.0%	6.1%

Sources and Notes:
[1]: S&P Credit Ratings from Research Insight.
[2]: Preferred ratings were assumed equal to debt ratings.
[3]: Schedule No. D5.6; Panel B, [10].
[4]: Schedule No. D5.4, [1].
[5]: Supporting Schedule #2 to Schedule No. D5.11, Panel C.
[6]: Schedule No. D5.4, [2].
[7]: Supporting Schedule #2 to Schedule No. D5.11, Panel B.
[8]: Schedule No. D5.4, [3].
[9]: VVC Effective Corporate Tax Rate.
[10]: ([3] x [4]) + ([5] x [6]) + {[7] x [8] x (1 - [9])}. A strikethrough indicates the utility was excluded from the full sample average calculation as a result of its cost of equity not exceeding its cost of debt by 100 basis points.

Schedule No. D5.8

DCF Cost of Equity at Vectren's Capital Structure

	Overall After -Tax Cost of Capital [1]	VVC Representative Base Deemed % Debt [2]	Representative Cost of A Rated Utility Debt [3]	VVC Representative Income Tax Rate [4]	VVC Representative Base Deemed % Equity [5]	Estimated Return on Equity [6]
Full Sample						
Simple DCF Quarterly	8.4%	49.4%	3.9%	21.0%	50.6%	13.7%
Multi-Stage DCF - Using Long-Term GDP Growth Forecast as the Perpetual Rate	6.3%	49.4%	3.9%	21.0%	50.6%	9.4%
Subsample						
Simple DCF Quarterly	7.5%	49.4%	3.9%	21.0%	50.6%	11.9%
Multi-Stage DCF - Using Long-Term GDP Growth Forecast as the Perpetual Rate	6.1%	49.4%	3.9%	21.0%	50.6%	9.1%

Sources and Notes:
[1]: Schedule No. D5.7; Panels A-B, [10].
[2]: VVC Assumed Capital Structure.
[3]: Based on an A rating. Yield from Bloomberg as of January 31, 2018.
[4]: VVC Effective Corporate Tax Rate.
[5]: VVC Assumed Capital Structure.
[6]: {[1] - ([2] x [3] x (1 - [4]))} / [5].

Schedule No. D5.9

Risk-Free Rates

[1]	Consensus 10-Year Forecast	3.40%
	U.S. Government Bond Yields	
[2]	20-Year	4.81%
[3]	10-Year	4.27%
[4]	Maturity Premium	0.54%
[5]	Consensus 10-Year Forecast Adjusted to 20-year Horizon	3.94%

Sources and Notes:

[1]: Bluechip Consensus Forecast in October 2017.

[2]-[3]: Supporting Schedule # 1 to Schedule No. D5.9. Averages of monthly bond yields from September 1992 through September 2017.

[4]: [2] - [3].

[5]: [1] + [4].

Schedule No. D5.10

Risk Positioning Cost of Equity of the Expanded Sample

l A: Scenario 1 - Long-Term Risk Free Rate of 4.14%, Long-Term Market Risk Premium of 6.

Company	Long-Term Risk-Free Rate [1]	Value Line Betas [2]	Long-Term Market Risk Premium [3]	CAPM Cost of Equity [4]	ECAPM (1.5%) Cost of Equity [5]
Atmos Energy	4.14%	0.70	6.94%	9.0%	9.4%
Chesapeake Utilities	4.14%	0.70	6.94%	9.0%	9.4%
ONE Gas Inc.	4.14%	0.70	6.94%	9.0%	9.4%
South Jersey Inds.	4.14%	0.85	6.94%	10.0%	10.3%
Southwest Gas	4.14%	0.80	6.94%	9.7%	10.0%
Spire Inc.	4.14%	0.70	6.94%	9.0%	9.4%
New Jersey Resources	4.14%	0.80	6.94%	9.7%	10.0%
Northwest Natural Gas	4.14%	0.70	6.94%	9.0%	9.4%
WGL Holdings Inc.	4.14%	0.80	6.94%	9.7%	10.0%
Average	4.14%	75.00%	6.94%	9.3%	9.7%
Subsample Average	4.14%	72.00%	6.94%	9.1%	9.6%

Sources and Notes:

[1]: Vilbert Direct Testimony.

[2]: Bloomberg as of January 31, 2018.

[3]: Vilbert Direct Testimony.

[4]: [1] + ([2] x [3]).

[5]: ([1] + 1.5%) + [2] x ([3] - 1.5%).

Schedule No. D5.10

Risk Positioning Cost of Equity of the Expanded Sample

l B: Scenario 2 - Long-Term Risk Free Rate of 3.94%, Long-Term Market Risk Premium of 7.

Company	Long-Term Risk-Free Rate [1]	Value Line Betas [2]	Long-Term Market Risk Premium [3]	CAPM Cost of Equity [4]	ECAPM (1.5%) Cost of Equity [5]
Atmos Energy	3.94%	0.70	7.94%	9.5%	9.9%
Chesapeake Utilities	3.94%	0.70	7.94%	9.5%	9.9%
ONE Gas Inc.	3.94%	0.70	7.94%	9.5%	9.9%
South Jersey Inds.	3.94%	0.85	7.94%	10.7%	10.9%
Southwest Gas	3.94%	0.80	7.94%	10.3%	10.6%
Spire Inc.	3.94%	0.70	7.94%	9.5%	9.9%
New Jersey Resources	3.94%	0.80	7.94%	10.3%	10.6%
Northwest Natural Gas	3.94%	0.70	7.94%	9.5%	9.9%
WGL Holdings Inc.	3.94%	0.80	7.94%	10.3%	10.6%
Average	3.94%	75.00%	7.94%	9.9%	10.3%
Subsample Average	3.94%	71.67%	7.94%	9.6%	10.1%

Sources and Notes:

[1]: Vilbert Direct Testimony.

[2]: Bloomberg as of January 31, 2018.

[3]: Vilbert Direct Testimony.

[4]: [1] + ([2] x [3]).

[5]: ([1] + 1.5%) + [2] x ([3] - 1.5%).

Schedule No. D5.11

Overall After-Tax Risk Positioning Cost of Capital of the Expanded Sample

Panel A: CAPM Cost of Equity Scenario 1 - Long-Term Risk Free Rate of 4.14%, Long-Term Market Risk Premium of 6.94%

Company	CAPM Cost of Equity [1]	ECAPM (1.5%) Cost of Equity [2]	5-Year Average Common Equity to Market Value Ratio [3]	Weighted - Average Cost of Preferred Equity [4]	5-Year Average Preferred Equity to Market Value Ratio [5]	Weighted-Average Cost of Debt [6]	5-Year Average Debt to Market Value Ratio [7]	VVC Representative Income Tax Rate [8]	Overall After-Tax Cost of Capital (CAPM) [9]	Overall After-Tax Cost of Capital (ECAPM 1.5%) [10]
Atmos Energy	9.0%	9.4%	63.5%	-	0.0%	3.95%	36.5%	21.0%	6.9%	7.1%
Chesapeake Utilities	9.0%	9.4%	73.2%	-	0.0%	-	26.8%	21.0%	NA	NA
ONE Gas Inc.	9.0%	9.4%	47.9%	-	0.0%	3.89%	22.1%	21.0%	5.0%	5.2%
South Jersey Inds.	10.0%	10.3%	61.9%	-	0.0%	4.19%	38.1%	21.0%	7.5%	7.6%
Southwest Gas	9.7%	10.0%	62.7%	-	0.0%	4.07%	37.3%	21.0%	7.3%	7.5%
Spire Inc.	9.0%	9.4%	56.6%	-	0.0%	3.89%	43.4%	21.0%	6.4%	6.7%
New Jersey Resources	9.7%	10.0%	74.3%	-	0.0%	-	25.7%	21.0%	NA	NA
Northwest Natural Gas	9.0%	9.4%	62.4%	-	0.0%	3.89%	37.6%	21.0%	6.8%	7.1%
WGL Holdings Inc.	9.7%	10.0%	69.6%	3.89%	0.7%	3.89%	29.7%	21.0%	7.7%	7.9%
Full Sample Average	9.3%	9.7%	63.6%	3.9%	0.1%	4.0%	33.0%	21.0%	6.8%	7.0%
Subsample Average	9.1%	9.5%	61.1%	-	0.0%	3.9%	33.9%	21.0%	6.5%	6.7%

Sources and Notes:

[1]: Schedule No. D5.10; Panel A, [4].
[2]: Schedule No. D5.10; Panel A, [5].
[3]: Schedule No. D5.4, [4].
[4]: Supporting Schedule #2 to Schedule No. D5.11
[5]: Schedule No. D5.4, [5].
[6]: Supporting Schedule #2 to Schedule No. D5.11, Pane
[7]: Schedule No. D5.4, [6].
[8]: VVC Effective Corporate Tax Rate
[9]: ([1] x [3]) + ([4] x [5]) + {[6] x [7] x (1 - [8])}.
[10]: ([2] x [3]) + ([4] x [5]) + {[6] x [7] x (1 - [8])}.

[9]-[10] A strikethrough indicates the utility was excluded from the full sample average calculation as a result of its cost of equity not exceeding its cost of debt by 100 basis points

Schedule No. D5.11

Overall After-Tax Risk Positioning Cost of Capital of the Expanded Sample

Panel B: CAPM Cost of Equity Scenario 2 - Long-Term Risk Free Rate of 3.94%, Long-Term Market Risk Premium of 7.94%

Company	CAPM Cost of Equity [1]	ECAPM (1.5%) Cost of Equity [2]	5-Year Average Common Equity to Market Value Ratio [3]	Weighted - Average Cost of Preferred Equity [4]	5-Year Average Preferred Equity to Market Value Ratio [5]	Weighted-Average Cost of Debt [6]	5-Year Average Debt to Market Value Ratio [7]	VVC Representative Income Tax Rate [8]	Overall After-Tax Cost of Capital (CAPM) [9]	Overall After-Tax Cost of Capital (ECAPM 1.5%) [10]
Atmos Energy	9.5%	9.9%	63.5%	-	0.0%	3.95%	36.5%	21.0%	7.2%	7.5%
Chesapeake Utilities	9.5%	9.9%	73.2%	-	0.0%	-	26.8%	21.0%	NA	NA
ONE Gas Inc.	9.5%	9.9%	47.9%	-	0.0%	3.89%	22.1%	21.0%	5.2%	5.4%
South Jersey Inds.	10.7%	10.9%	61.9%	-	0.0%	4.19%	38.1%	21.0%	7.9%	8.0%
Southwest Gas	10.3%	10.6%	62.7%	-	0.0%	4.07%	37.3%	21.0%	7.6%	7.8%
Spire Inc.	9.5%	9.9%	56.6%	-	0.0%	3.89%	43.4%	21.0%	6.7%	7.0%
New Jersey Resources	10.3%	10.6%	74.3%	-	0.0%	-	25.7%	21.0%	NA	NA
Northwest Natural Gas	9.5%	9.9%	62.4%	-	0.0%	3.89%	37.6%	21.0%	7.1%	7.4%
WGL Holdings Inc.	10.3%	10.6%	69.6%	3.89%	0.7%	3.89%	29.7%	21.0%	8.1%	8.3%
Full Sample Average	9.9%	10.3%	63.6%	3.9%	0.1%	4.0%	33.0%	21.0%	7.1%	7.3%
Subsample Average	9.6%	10.1%	61.1%	-	0.0%	3.9%	33.9%	21.0%	6.8%	7.0%

Sources and Notes:
[1]: Schedule No. D5.10; Panel B, [4].
[2]: Schedule No. D5.10; Panel B, [5].
[3]: Schedule No. D5.4, [4].
[4]: Supporting Schedule #2 to Schedule No. D5.11
[5]: Schedule No. D5.4, [5].
[6]: Supporting Schedule #2 to Schedule No. D5.11, Pane
[7]: Schedule No. D5.4, [6].
[8]: VVC Effective Corporate Tax Rate
[9]: ([1] x [3]) + ([4] x [5]) + {[6] x [7] x (1 - [8])}.
[10]: ([2] x [3]) + ([4] x [5]) + {[6] x [7] x (1 - [8])}.
[9]-[10] A strikethrough indicates the utility was excluded from the full sample average calculation as a result of its cost of equity not exceeding its cost of debt by 100 basis points

Schedule No. D5.12

Risk Positioning Cost of Equity at Vectren's Capital Structure

	Overall After-Tax Cost of Capital (Scenario 1) [1]	Overall After-Tax Cost of Capital (Scenario 2) [2]	VVC Representative Base Deemed % Debt [3]	Representative Cost of A-Rated Utility Debt [4]	VVC Representative Income Tax Rate [5]	VVC Representative Base Deemed % Equity [6]	Estimated Return on Equity (Scenario 1) [7]	Estimated Return on Equity (Scenario 2) [8]
Full Sample:								
CAPM	6.8%	7.1%	49.4%	3.9%	21.0%	50.6%	10.4%	11.1%
ECAPM (1.50%)	7.0%	7.3%	49.4%	3.9%	21.0%	50.6%	10.8%	11.5%
Subsample:								
CAPM	6.5%	6.8%	49.4%	3.9%	21.0%	50.6%	9.8%	10.4%
ECAPM (1.50%)	6.7%	7.0%	49.4%	3.9%	21.0%	50.6%	10.3%	10.9%

Sources and Notes:

[1]: Schedule No. D5.11; Panel A, [9] - [10]. Scenario 1: Long-Term Risk Free Rate of 4.14%, Long-Term Market Risk Premium of 6.94%.

[2]: Schedule No. D5.11; Panel B, [9] - [10]. Scenario 2: Long-Term Risk Free Rate of 3.94%, Long-Term Market Risk Premium of 7.94%.

[3]: VVC Assumed Capital Structure.

[4]: Based on a A rating. Yield from Bloomberg as of January 31, 2018.

[5]: VVC Effective Corporate Tax Rate.

[6]: VVC Assumed Capital Structure.

[7]: {[1] - ([3] x [4] x (1 - [5])}/ [6].

[8]: {[2] - ([3] x [4] x (1 - [5]))}/ [6].

Schedule No. D5.13

Hamada Adjustment to Obtain Unlevered Asset Beta

Company	Value Line Betas [1]	Debt Beta [2]	5-Year Average Common Equity to Market Value Ratio [3]	5-Year Average Preferred Equity to Market Value Ratio [4]	5-Year Average Debt to Market Value Ratio [5]	VVC Representative Income Tax Rate [6]	Asset Beta: Without Taxes [7]	Asset Beta: With Taxes [8]
Atmos Energy	0.70	0.06	63.5%	0.0%	36.5%	21.0%	0.47	0.50
Chesapeake Utilities	0.70	n/a	73.2%	0.0%	26.8%	21.0%	n/a	n/a
ONE Gas Inc.	0.70	0.05	47.9%	0.0%	22.1%	21.0%	0.35	0.53
South Jersey Inds.	0.85	0.10	61.9%	0.0%	38.1%	21.0%	0.56	0.60
Southwest Gas	0.80	0.08	62.7%	0.0%	37.3%	21.0%	0.53	0.57
Spire Inc.	0.70	0.05	56.6%	0.0%	43.4%	21.0%	0.42	0.45
New Jersey Resources	0.80	n/a	74.3%	0.0%	25.7%	21.0%	n/a	n/a
Northwest Natural Gas	0.70	0.05	62.4%	0.0%	37.6%	21.0%	0.46	0.49
WGL Holdings Inc.	0.80	0.05	69.6%	0.7%	29.7%	21.0%	0.57	0.61
Full Sample Average	0.75	0.06	63.6%	0.00	33.0%	21.0%	0.48	0.54
Subsample Average	0.72	0.06	61.1%	0.00	33.9%	21.0%	0.44	0.51

Sources and Notes:

[1]: Supporting Schedule # 1 to Schedule No. D5.10, [1].
[2]: Supporting Schedule #1 to Schedule No. D5.13, [7].
[3]: Schedule No. D5.4, [4].
[4]: Schedule No. D5.4, [5].
[5]: Schedule No. D5.4, [6].
[6]: VVC Effective Corporate Tax Rate
[7]: [1]*[3] + [2]*([4] + [5]).
[8]: {[1]*[3] + [2]*([4]+[5]*(1-[6]))} / {[3] + [4] + [5]*(1 -[6])}.

Schedule No. D5.14

Expanded Sample Average Asset Beta Relevered at Vectren's Capital Structure

	Asset Beta [1]	Assumed Debt Beta [2]	VVC Representative Base Deemed % Debt [3]	VVC Representative Income Tax Rate [4]	VVC Representative Base Deemed % Equity [5]	Estimated Equity Beta [6]
Full Sample:						
Asset Beta Without Taxes	0.48	0.05	49.4%	21.0%	50.6%	0.90
Asset Beta With Taxes	0.54	0.05	49.4%	21.0%	50.6%	0.91
Subsample:						
Asset Beta Without Taxes	0.44	0.05	49.4%	21.0%	50.6%	0.83
Asset Beta With Taxes	0.51	0.05	49.4%	21.0%	50.6%	0.86

Sources and Notes:

[1]: Schedule No. D5.13, [7] - [8].

[2]: Debt Beta estimate for A-rated entities.Corporate Finance, Berk and Demarzo, Second Edition, p. 389.

[3]: VVC Assumed Capital Structure.

[4]: VVC Effective Corporate Tax Rate.

[5]: VVC Assumed Capital Structure.

[6]: [1] + [3]/[5]*([1] - [2]) without taxes, [1] + [3]*(1 - [4])/[5]*([1] - [2]) with taxes.

Schedule No. D5.15

Risk-Positioning Cost of Equity using Hamada-Adjusted Betas

Panel A: Scenario 1 - Long-Term Risk Free Rate of 4.14%, Long-Term Market Risk Premium of 6.94%

Company	Long-Term Risk-Free Rate [1]	Hamada Adjusted Equity Betas [2]	Long-Term Market Risk Premium [3]	CAPM Cost of Equity [4]	ECAPM (1.5%) Cost of Equity [5]
Full Sample:				capmlt	ecapmlt2
Asset Beta Without Taxes	4.14%	0.90	6.94%	10.4%	10.5%
Asset Beta With Taxes	4.14%	0.91	6.94%	10.5%	10.6%
Subsample:					
Asset Beta Without Taxes	4.14%	0.83	6.94%	9.9%	10.1%
Asset Beta With Taxes	4.14%	0.86	6.94%	10.1%	10.3%

Sources and Notes:
[1]: Vilbert Direct Testimony.
[2]: Schedule No. D5.14, [6].
[3]: Vilbert Direct Testimony.
[4]: [1] + ([2] x [3]).
[5]: ([1] + 1.5%) + [2] x ([3] - 1.5%).

Schedule No. D5.15

Risk-Positioning Cost of Equity using Hamada-Adjusted Betas

Panel B: Scenario 2 - Long-Term Risk Free Rate of 3.94%, Long-Term Market Risk Premium of 7.94%

Company	Long-Term Risk-Free Rate [1]	Hamada Adjusted Equity Betas [2]	Long-Term Market Risk Premium [3]	CAPM Cost of Equity [4]	ECAPM (1.5%) Cost of Equity [5]
Full Sample:				capmlt	ecapmlt2
Asset Beta Without Taxes	3.94%	0.90	7.94%	11.1%	11.2%
Asset Beta With Taxes	3.94%	0.91	7.94%	11.2%	11.3%
Subsample:					
Asset Beta Without Taxes	3.94%	0.83	7.94%	10.5%	10.8%
Asset Beta With Taxes	3.94%	0.86	7.94%	10.8%	11.0%

Sources and Notes:
[1]: Vilbert Direct Testimony.
[2]: Schedule No. D5.14, [6].
[3]: Vilbert Direct Testimony.
[4]: [1] + ([2] x [3]).
[5]: ([1] + 1.5%) + [2] x ([3] - 1.5%).

Schedule No. D5.16
Risk Premiums Determined by Relationship Between Authorized ROEs[1] and Long-term Treasury Bond Rates During the Period 1990-2017

Formula: Risk Premium = A_0 + (A_1 x Treasury bond Rate)

R Squared	0.8479
Estimate of intercept (A_0)	8.407%
Estimate of slope (A_1)	-0.5611

Equity Cost Estimate for Gas LDC		Predicted Risk Premium		Expected Treasury Bond Rate[2]	
10.2%	=	6.08%	+	4.14%	[3]
10.1%	=	6.20%	+	3.94%	[4]

Sources and Notes:
[1]: Authorized ROE Data sourced from SNL Financial.
[2]: Blue Chip consensus forecast 2019 10-yr T-bill Yield plus maturity premium
[3]: Estimate with expected treasury bond rate normalized with 0.20% utility yield spread adjustment
[4]: Estimate without treasury bond rate normalization.
See regression results for derivation of regression coefficients A_0 and A_1.

Schedule No. D5.17

Academic Literature on Financial Risk Adjustments

THE EFFECT OF THE FIRM'S CAPITAL STRUCTURE ON THE SYSTEMATIC RISK OF COMMON STOCKS

ROBERT S. HAMADA*

I. INTRODUCTION

ONLY RECENTLY has there been an interest in relating the issues historically associated with corporation finance to those historically associated with investment and portfolio analyses. In fact, rigorous theoretical attempts in this direction were made only since the capital asset pricing model of Sharpe [13], Lintner [6], and Mossin [11], itself an extension of the Markowitz [7] portfolio theory. This study is one of the first empirical works consciously attempting to show and test the relationships between the two fields. In addition, differences in the observed systematic or nondiversifiable risk of common stocks, β, have never really been analyzed before by investigating some of the underlying differences in the firms.

In the capital asset pricing model, it was demonstrated that the efficient set of portfolios to any individual investor will always be some combination of lending at the risk-free rate and the "market portfolio," or borrowing at the risk-free rate and the "market portfolio." At the same time, the Modigliani and Miller (MM) propositions [9, 10] on the effect of corporate leverage are well known to the students of corporation finance. In order for their propositions to hold, personal leverage is required to be a perfect substitute for corporate leverage. If this is true, then corporate borrowing could substitute for personal borrowing in the capital asset pricing model as well.

Both in the pricing model and the MM theory, borrowing, from whatever source, while maintaining a fixed amount of equity, increases the risk to the investor. Therefore, in the mean-standard deviation version of the capital asset pricing model, the covariance of the asset's rate of return with the market portfolio's rate of return (which measures the nondiversifiable risk of the asset—the proxy β will be used to measure this) should be greater for the stock of a firm with a higher debt-equity ratio than for the stock of another firm in the same risk-class with a lower debt-equity ratio.[1]

This study, then, has a number of purposes. First, we shall attempt to link empirically corporation finance issues with portfolio and security analyses through the effect of a firm's leverage on the systematic risk of its common

* Graduate School of Business, University of Chicago, currently visiting at the Graduate School of Business Administration, University of Washington. The research assistance of Christine Thomas and Leon Tsao is gratefully acknowledged. This paper has benefited from the comments made at the Finance Workshop at the University of Chicago, and especially those made by Eugene Fama. Remaining errors are due solely to the author.

1. This very quick summary of the theoretical relationship between what is known as corporation finance and the modern investment and portfolio analyses centered around the capital asset pricing model is more thoroughly presented in [5], along with the necessary assumptions required for this relationship.

stock. Then, we shall attempt to test the MM theory, or at least provide another piece of evidence on this long-standing controversial issue. This test will not rely on an explicit valuation model, such as the MM study of the electric utility industry [8] and the Brown study of the railroad industry [2]. A procedure using systematic risk measures (β s) has been worked out in this paper for this purpose.

If the MM theory is validated by this procedure, then the final purpose of this study is to demonstrate a method for estimating the cost of capital of individual firms to be used by them for scale-changing or nondiversifying investment projects. The primary component of any firm's cost of capital is the capitalization rate for the firm if the firm had no debt and preferred stock in its capital structure. Since most firms do have fixed commitment obligations, this capitalization rate (we shall call it $E(R_A)$; MM denote it ρ^τ) is unobservable. But if the MM theory and the capital asset pricing model are correct, then it is possible to estimate $E(R_A)$ from the systematic risk approach for individual firms, even if these firms are members of a one-firm risk-class.[2]

With this statement of the purposes for this study, we shall, in Section II, discuss the alternative general procedures that are possible for estimating the effect of leverage on systematic risk and select the most feasible ones. The results are presented in Section III. And finally, tests of the MM versus the traditional theories of corporation finance are presented in Section IV.

II. Some Possible Procedures and the Selected Estimating Relationships

There are at least four general procedures that can be used to estimate the effect of the firm's capital structure on the systematic risk of common stocks. The first is the MM valuation model approach. By estimating ρ^τ with an explicit valuation model as they have for the electric utility industry, it is possible to relate this ρ^τ with the use of the capital asset pricing model to a nonleveraged systematic risk measure, ${}_A\beta$. Then the difference between the observed common stock's systematic risk (which we shall denote ${}_B\beta$) and ${}_A\beta$ would be due solely to leverage. But the difficulties of this approach for all firms are many.

The MM valuation model approach requires the specification, in advance, of risk-classes. All firms in a risk-class are then assumed to have the same ρ^τ—the capitalization rate for an all-common equity firm. Unfortunately, there must be enough firms in a risk-class so that a cross-section analysis will yield statistically significant coefficients. There may not be many more risk-classes (with enough observations) now that the electric utility and railroad industries have been studied. In addition, the MM approach requires estimating expected asset earnings and estimating the capitalized growth potential implicit in stock prices. If it is possible to consider growth and expected earnings without having

2. It is, in fact, this last purpose of making applicable and practical some of the implications of the capital asset pricing model for corporation finance issues that provided the initial motivation for this paper. In this context, if one is familiar with the fair rate of return literature for regulated utilities, for example, an industry where debt is so prevalent, adjusting correctly for leverage is not frequently done and can be very critical.

to specify their exact magnitude at a specific point in time, considerable difficulty and possible measurement errors will be avoided.

The second approach is to run a regression between the observed systematic risk of a stock and a number of accounting and leverage variables in an attempt to explain this observed systematic risk. Unfortunately, without a theory, we do not know which variables to include and which variables to exclude and whether the relationship is linear, multiplicative, exponential, curvilinear, etc. Therefore, this method will also not be used.

A third approach is to measure the systematic risk before and after a new debt issue. The difference can then be attributed to the debt issue directly. An attractive feature of this procedure is that a good estimate of the market value of the incremental debt issue can be obtained. A number of disadvantages, unfortunately, are associated with this direct approach. The difference in the systematic risk may be due not only to the additional debt, but also to the reason the debt was issued. It may be used to finance a new investment project, in which case the project's characteristics will also be reflected in the new systematic risk measure. In addition, the new debt issue may have been anticipated by the market if the firm had some long-run target leverage ratio which this issue will help maintain; conversely, the market may not fully consider the new debt issue if it believes the increase in leverage is only temporary. For these reasons, this seemingly attractive procedure will not be employed.

The last approach, which will be used in this study, is to assume the validity of the MM theory from the outset. Then the observed rate of return of a stock can be adjusted to what *it would have been* over the same time period had the firm no debt and preferred stock in its capital structure. The difference between the observed systematic risk, ${}_B\beta$, and the systematic risk for this adjusted rate of return time series, ${}_A\beta$, can be attributed to leverage, if the MM theory is correct. The final step, then, is to test the MM theory.

To discuss this more specifically, consider the following relationship for the dollar return to the common shareholder from period $t-1$ to t:

$$(X-I)_t(1-\tau)_t - p_t + \Delta G_t = d_t + cg_t \tag{1}$$

where X_t represents earnings before taxes, interest, and preferred dividends and is assumed to be unaffected by fixed commitment obligations; I_t represents interest and other fixed charges paid during the period; τ is the corporation income tax rate; p_t is the preferred dividends paid; ΔG_t represents the change in capitalized growth over the period; and d_t and cg_t are common shareholder dividends and capital gains during the period, respectively.

Equation (1) relates the corporation finance types of variables with the market holding period return important to the investors. The first term on the left-hand-side of (1) is profits after taxes and after interest which is the earnings the common and preferred shareholders receive on their investment for the period. Subtracting out p_t leaves us with the earnings the common shareholder would receive from currently-held assets.

To this must be added any change in capitalized growth since we are trying to explain the common shareholder's market holding period dollar return. ΔG_t

must be added for growth firms to the current period's profits from existing assets since capitalized growth opportunities of the firm—future earnings from new assets over and above the firm's cost of capital which are already reflected in the stock price at $(t-1)$—should change over the period and would accrue to the common shareholder. Assuming shareholders at the start of the period estimated these growth opportunities on average correctly, the expected value of ΔG_t would not be zero, but should be positive. For example, consider growth opportunities five years from now which yield more than the going rate of return and are reflected in today's stock price. These growth opportunities will become one year closer to fruition at time t than at time $t-1$ so that their present value would become larger. ΔG_t then represents this increase in the present value of these future opportunities simply because it is now four years away rather than five.[3]

Since the systematic risk of a common stock is:

$$ {}_B\beta = \frac{\text{cov}\,(R_{B_t}, R_{M_t})}{\sigma^2(R_{M_t})} \tag{2} $$

where R_{B_t} is the common shareholder's rate of return and R_{M_t} is the rate of return on the market portfolio, then substitution of (1) into (2) yields:

$$ {}_B\beta = \frac{\text{cov}\left[\dfrac{(X-I)(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{2a} $$

where $S_{B_{t-1}}$ denotes the market value of the common stock at the beginning of the period.

The systematic risk for the same firm over the same period *if* there were no debt and preferred stock in its capital structure is:

$$ {}_A\beta = \frac{\text{cov}(R_{A_t}, R_{M_t})}{\sigma^2(R_{M_t})} = \frac{\text{cov}\left[\dfrac{X(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{3} $$

where R_{A_t} and $S_{A_{t-1}}$ represent the rate of return and the market value, respectively, to the common shareholder if the firm had no debt and preferred stock. From (3), we can obtain:

$$ {}_A\beta S_{A_{t-1}} = \frac{\text{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} \tag{3a} $$

3. Continual awareness of the difficulties of estimating capitalized growth, or changes in growth, especially in conjunction with leverage considerations, for purposes such as valuation or cost of capital is a characteristic common to students of corporation finance. This is the reason for the emphasis on growth in this paper and for presenting a method to neutralize for differences in growth when comparing rates of return.

Next, by expanding and rearranging (2a), we have:

$$_B\beta S_{B_{t-1}} = \frac{\mathrm{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\mathrm{cov}\,[I(1-\tau)_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\mathrm{cov}\,(p_t, R_{M_t})}{\sigma^2(R_{M_t})} \tag{2b}$$

If we assume as an empirical approximation that interest and preferred dividends have negligible covariance with the market, at least relative to the (pure equity) common stock's covariance, then substitution of the LHS of (3a) into the RHS of (2b) yields:[4]

$$_B\beta S_{B_{t-1}} = {}_A\beta S_{A_{t-1}} \tag{4}$$

or

$$_A\beta = \left(\frac{S_B}{S_A}\right)_{t-1} {}_B\beta \tag{4a}$$

Because $S_{A_{t-1}}$, the market value of common stock *if* the firm had no debt and preferred stock, is not observable since most firms do have debt and/or preferred stock, a theory is required in order to measure what this quantity *would have been* at $t-1$. The MM theory [10] will be employed for this purpose, that is:

$$S_{A_{t-1}} = (V - \tau D)_{t-1}. \tag{5}$$

Equation (5) indicates that if the Federal government tax subsidy for debt financing, τD, where D is the market value of debt, is subtracted from the observed market value of the firm, V_{t-1} (where V_{t-1} is the sum of S_B, D and the observed market value of preferred), then the market value of an unleveraged firm is obtained. Underlying (5) is the assumption that the firm is near its target leverage ratio so that no more or no less debt subsidy is capitalized already into the observed stock price. The conditions under which this MM relationship hold are discussed carefully in [4].

It is at this point that problems in obtaining satisfactory estimates of $_A\beta$ develop, since (4) theoretically holds only for the next period. As a practical matter, the accepted, and seemingly acceptable, method of obtaining estimates of a stock's systematic risk, $_B\beta$, is to run a least squares regression between a stock's and market portfolio's *historical* rates of return. Using past data for $_B\beta$, it is not clear which *period's* ratio of market values to apply in (4a) to estimate the firm's systematic risk, $_A\beta$. There would be no problem if the market value ratios of debt to equity and preferred stock to equity remained relatively stable over the past for each firm, but a cursory look at these data reveals that this is not true for the large majority of firms in our sample. Should we use the market value ratio required in (4a) that was observed at the start of our regression period, at the end of our regression period, or some kind of average over the period? In addition, since these different observed ratios will give us different estimates for $_A\beta$, it is not clear, without some criterion, how we should select from among the various estimates.

4. This general method of arriving at (4) was suggested by the comments of William Sharpe, one of the discussants of this paper at the annual meeting. A much more cumbersome and less general derivation of (4) was in the earlier version.

It is for this purpose—to obtain a standard—that a more cumbersome and more data demanding approach to obtain estimates of $_A\beta$ is suggested. Given the large fluctuations in market leverage ratios, intuitively it would appear that the firm's risk is more stable than the common stock's risk. In that event, a leverage-free rate of return time series for each firm should be derived and the market model applied to this time series directly. In this manner, the beta coefficient would give us a *direct* estimate of $_A\beta$ which can then be used as a criterion to determine if any of the market value ratios discussed above can be applied to (4a) successfully.

For this purpose, the "would-have-been" rate of return for the common stock if the firm had no debt and preferred is:

$$R_{A_t} = \frac{X_t(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}. \tag{6}$$

The numerator of (6) can be rearranged to be:

$$X_t(1-\tau)_t + \Delta G_t \equiv [(X-I)_t(1-\tau)_t - p_t + \Delta G_t] + p_t + I_t(1-\tau)_t.$$

Substituting (1):

$$X_t(1-\tau)_t + \Delta G_t = [d_t + cg_t] + p_t + I_t(1-\tau)_t.$$

Therefore, (6) can be written as:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{S_{A_{t-1}}}. \tag{7}$$

Since $S_{A_{t-1}}$ is unobservable for the firms with leverage, the MM theory, equation (5), will be employed; then:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{(V-\tau D)_{t-1}}. \tag{8}$$

The observed rate of return on the common stock is, of course:

$$R_{B_t} = \frac{(X-I)_t(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}} = \frac{d_t + cg_t}{S_{B_{t-1}}}. \tag{9}$$

Equation (8) is the rate of return to the common shareholder of the same firm and over the same period of time as (9). However, in (8) there are the underlying assumptions that the firm never had any debt and preferred stock and that the MM theory is correct; (9) incorporates the exact amount of debt and preferred stock that the firm actually did have over this time period and no leverage assumption is being made. Both (8) and (9) are now in forms where they can be measured with available data. One can note that it is unnecessary to estimate the change in growth, or earnings from current assets, since these should be captured in the market holding period return, $d_t + cg_t$.

Using CRSP data for (9) and both CRSP and Compustat data for the components of (8), a time series of yearly R_{A_t} and R_{B_t} for $t = 1948\text{-}1967$ were derived for 304 different firms. These 304 firms represent an exhaustive sample of the firms with complete data on both tapes for all the years.

A number of "market model" [1, 12] variants were then applied to these data. For each of the 304 firms, the following regressions were run:

$$R_{Ait} = {}_A\alpha_i + {}_A\beta_i R_{Mt} + {}_A\epsilon_{it} \tag{10a}$$

$$R_{Bit} = {}_B\alpha_i + {}_B\beta_i R_{Mt} + {}_B\epsilon_{it} \tag{10b}$$

$$\ln(1 + R_{Ait}) = {}_{AC}\alpha_i + {}_{AC}\beta_i \ln(1 + R_{M_t}) + {}_{AC}\epsilon_{it} \tag{10c}$$

$$\ln(1 + R_{Bit}) = {}_{BC}\alpha_i + {}_{BC}\beta_i \ln(1 + R_{M_t}) + {}_{BC}\epsilon_{it} \tag{10d}$$

$$i = 1, 2, \ldots, 304$$
$$t = 1948\text{-}1967$$

where R_{Mt} is the observed NYSE arithmetic stock market rate of return with dividends reinvested, α_i and β_i are constants for each firm-regression, and the usual conditions are assumed for the properties of the disturbance terms, ϵ_{it}. Equations (10c) and (10d) are the continuously-compounded rate of return versions of (10a) and (10b), respectively.[5]

III. The Results

An abbreviated table of the regression results for each of the four variants, equations (10a)-(10d), summarized across the 304 firms is shown in Table 1.

The first column designated "mean" is the average of the statistic (indicated by the rows) over all 304 firms. Therefore, the mean ${}_A\hat{\alpha}$ of 0.0221 is the intercept term of equation (10a) averaged over 304 different firm-regressions. The second and third columns give the deviation measures indicated, of the 304 point estimates of, say, ${}_A\hat{\alpha}$. The mean standard error of estimate in the last column is the average over 304 firms of the individual standard errors of estimate.

The major conclusion drawn from Table 1 is the following mean β comparisons:

$${}_B\hat{\beta} > {}_A\hat{\beta}, \text{ i.e., } 0.9190 > 0.7030$$
$${}_{BC}\hat{\beta} > {}_{AC}\hat{\beta}, \text{ i.e., } 0.9183 > 0.7263.$$

The directional results of these betas, assuming the validity of the MM theory, are not imperceptible and clearly are not negligible differences from the investor's point of view. This is obtained in spite of all the measurement and data problems associated with estimating a time series of the RHS of (8) for

5. Because the R_{M_t} used in equations (10) is defined as the observed stock market return, and since adjusting for capital structure is the major purpose of this exercise, it was decided that the same four regressions should be replicated on a leverage-adjusted stock market rate of return. The major reason for this additional adjustment is the belief that the rates of return over time and their relationship with the market are more stable when we can abstract from all changes in leverage and get at the underlying risk of all firms.

For the 221 firms (out of the total 304) whose fiscal years coincide with the calendar year, average values for the components of the RHS of (8) were obtained for each year so that R_{M_t} could be adjusted in the same way as for the individual firms—a yearly time series of stock market rates of return, if all the firms on the NYSE had no debt and no preferred in their capital structure, was derived. The results, when using this adjusted market portfolio rate of return time series, were not very different from the results of equations (10), and so will not be reported here separately.

TABLE 1
SUMMARY RESULTS OVER 304 FIRMS OF EQUATIONS (10a)-(10d)

	Mean	Mean Absolute Deviation*	Standard Deviation	Mean Standard Error of Estimate
${}_{A}\hat{\alpha}$	0.0221	0.0431	0.0537	0.0558
${}_{A}\hat{\beta}$	0.7030	0.2660	0.3485	0.2130
${}_{A}\hat{R}^2$	0.3799	0.1577	0.1896	
${}_{A}\hat{\rho}$	0.0314			
${}_{B}\hat{\alpha}$	0.0187	0.0571	0.0714	0.0720
${}_{B}\hat{\beta}$	0.9190	0.3550	0.4478	0.2746
${}_{B}\hat{R}^2$	0.3864	0.1578	0.1905	
${}_{B}\hat{\rho}$	0.0281			
${}_{AC}\hat{\alpha}$	0.0058	0.0427	0.0535	0.0461
${}_{AC}\hat{\beta}$	0.7263	0.2700	0.3442	0.2081
${}_{AC}\hat{R}^2$	0.3933	0.1586	0.1909	
${}_{AC}\hat{\rho}$	0.0268			
${}_{BC}\hat{\alpha}$	−0.0052	0.0580	0.0729	0.0574
${}_{BC}\hat{\beta}$	0.9183	0.3426	0.4216	0.2591
${}_{BC}\hat{R}^2$	0.4012	0.1602	0.1922	
${}_{BC}\hat{\rho}$	0.0262			

* Defined as: $\dfrac{\sum_{i=1}^{N} |x_i - \bar{x}|}{N}$, where $N = 304$. $\hat{\rho}$ = first order serial correlation coefficient.

each firm. One of the reasons for the "traditional" theory position on leverage is precisely this point—that small and reasonable amounts of leverage cannot be discerned by the market. In fact, if the MM theory is correct, leverage has explained as much as, roughly, 21 to 24 per cent of the value of the mean β.

We can also note that if the covariance between the asset and market rates of return, as well as the market variance, was constant over time, then the systematic risk from the market model is related to the expected rate of return by the capital asset pricing model. That is:

$$E(R_{A_t}) = R_{F_t} + {}_{A}\beta[E(R_{M_t}) - R_{F_t}] \tag{11a}$$

$$E(R_{B_t}) = R_{F_t} + {}_{B}\beta[E(R_{M_t}) - R_{F_t}] \tag{11b}$$

Equation (11a) indicates the relationship between the expected rate of return for the common stock shareholder of a debt-free and preferred-free firm, to the systematic risk, ${}_{A}\beta$, as obtained in regressions (10a) or (10c). The LHS of (11a) is the important $\rho\tau$ for the MM cost of capital. The MM theory [9, 10] also predicts that shareholder expected yield must be higher (for the same real firm) when the firm has debt than when it does not. Financial risk is greater, therefore, shareholders require more expected return. Thus, $E(R_{B_t})$ must be greater than $E(R_{A_t})$. In order for this MM prediction to be true, from (11a) and (11b) it can be observed that ${}_{B}\beta$ must be greater than ${}_{A}\beta$, which is what we obtained.

Using the results underlying Table 1, namely the firm and stock betas, as the criterion for selecting among the possible observed market value ratios that can be used, if any, for (4), the following cross-section regressions were run:

$$({}_{B}\beta)_i = a_1 + b_1 \left(\frac{S_A}{S_B}\, {}_{A}\beta \right)_i + u_{1i} \qquad i = 1, 2, \ldots, 102 \qquad (12a)$$

$$({}_{BC}\beta)_i = a_2 + b_2 \left(\frac{S_A}{S_B}\, {}_{AC}\beta \right)_i + u_{2i} \qquad i = 1, 2, \ldots, 102 \qquad (12b)$$

$$({}_{A}\beta)_i = a_3 + b_3 \left(\frac{S_B}{S_A}\, {}_{B}\beta \right)_i + u_{3i} \qquad i = 1, 2, \ldots, 102 \qquad (13a)$$

$$({}_{AC}\beta)_i = a_4 + b_4 \left(\frac{S_B}{S_A}\, {}_{BC}\beta \right)_i + u_{4i} \qquad i = 1, 2, \ldots, 102 \qquad (13b)$$

Because the preferred stock market values were not as reliable as debt, only the 102 firms (out of 304) that did not have preferred in any of the years were used. The test for the adequacy of this alternative approach, equation (4), to adjust the systematic risk of common stocks for the underlying firm's capital structure, is whether the intercept term, a, is equal to zero, and the slope coefficient, b, is equal to one in the above regressions (as well as, of course, a high R^2)—these requirements are implied by (4). The results of this test would also indicate whether future "market model" studies that only use common stock rates of return without adjusting, or even noting, for the firm's debt-equity ratio will be adequate. The total firm's systematic risk may be stable (as long as the firm stays in the same risk-class), whereas the common stock's systematic risk may not be stable merely because of unanticipated capital structure changes—the data underlying Table 3 indicate that there were very few firms which did not have major changes in their capital structure over the twenty years studied.

The results of these regressions, when using the average S_A and average S_B over the twenty years for each firm, are shown in the first column panel of Table 2. These regressions were then replicated twice, first using the December 31, 1947 values of S_{A_i} and S_{B_i} instead of the twenty-year average for each firm, and then substituting the December 31, 1966 values of S_{A_i} and S_{B_i} for the 1947 values. These results are in the second and third panels of Table 2.[6]

From the first panel of Table 2, it appears that this alternative approach via (4a) for adjusting the systematic risk for the firm's leverage is quite

6. The point should be made that we are not merely regressing a variable on itself in (12) and (13). (12a) and (12b) can be interpreted as correlating the ${}_{B}\beta_i$ obtained from (10b) and (10d)—the LHS variable in (12a) and (12b)—against the ${}_{B}\beta_i$ obtained from rearranging (4)—the RHS variable in (12a) and (12b)—to determine whether the use of (4) is as good a means of obtaining ${}_{B}\beta_i$ as the direct way via the equations (10). We would be regressing a variable on itself only if the ${}_{A}\beta_i$ were calculated using (4a), and then the ${}_{A}\beta_i$ thus obtained, inserted into (12a) and (12b).

Instead, we are obtaining ${}_{A}\beta_i$ using the MM model in *each* of the twenty years so that a leverage-adjusted 20 year time series of R_{A_i} is derived. Of course, if there were no data nor measurement problems, and if the debt-to-equity ratio were perfectly stable over this twenty year period for each firm, then we should obtain perfect correlation in (12a) and (12b), with a = 0 and b = 1, as (4) would be an identity.

TABLE 2

RESULTS FOR THE EQUATIONS (12a), (12b), (13a), AND (13b)*

	Using 20-Year Average for $\left(\frac{S_A}{S_B}\right)_i$			Using 1947 Value for $\left(\frac{S_A}{S_B}\right)_i$			Using 1966 Value for $\left(\frac{S_A}{S_B}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (12a)	−0.022 (0.021)	1.062 (0.021)	0.962	0.150 (0.048)	0.842 (0.045)	0.781	0.085 (0.041)	0.905 (0.038)	0.849
	constant suppressed	1.042 (0.009)	0.962	constant suppressed	0.966 (0.021)	0.781	constant suppressed	0.976 (0.017)	0.849
Eq. (12b)	−0.003 (0.013)	1.016 (0.013)	0.984	0.159 (0.047)	0.816 (0.044)	0.773	0.124 (0.037)	0.843 (0.034)	0.859
	constant suppressed	1.014 (0.005)	0.984	constant suppressed	0.952 (0.019)	0.773	constant suppressed	0.947 (0.015)	0.859

	Using 20-Year Average for $\left(\frac{S_B}{S_A}\right)_i$			Using 1947 Value for $\left(\frac{S_B}{S_A}\right)_i$			Using 1966 Value for $\left(\frac{S_B}{S_A}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (13a)	0.030 (0.016)	0.931 (0.017)	0.969	0.112 (0.028)	0.843 (0.030)	0.888	0.080 (0.027)	0.898 (0.030)	0.902
	constant suppressed	0.960 (0.007)	0.969	constant suppressed	0.948 (0.015)	0.888	constant suppressed	0.976 (0.014)	0.902
Eq. (13b)	0.007 (0.010)	0.979 (0.011)	0.988	0.119 (0.026)	0.852 (0.028)	0.902	0.063 (0.026)	0.942 (0.029)	0.911
	constant suppressed	1.004 (0.012)	0.911	constant suppressed	0.967 (0.013)	0.902	constant suppressed	1.005 (0.012)	0.911

* Standard error in parentheses.

satisfactory (at least with respect to our sample of firms and years) only if long-run averages of S_A and S_B are used. The second and third panels indicate that the equations (8) and (10) procedure is markedly superior when only one year's market value ratio is used as the adjustment factor. The annual debt-to-equity ratio is much too unstable for this latter procedure.

Thus, when forecasting systematic risk is the primary objective—for example, for portfolio decisions or for estimating the firm's cost of capital to apply to prospective projects—a long-run forecasted leverage adjustment is required. Assuming the firm's risk is more stable than the common stock's risk,[7] and if there is some reason to believe that a better forecast of the firm's future leverage can be obtained than using simply a past year's (or an average of past years') leverage, it should be possible to improve the usual extrapolation forecast of a stock's systematic risk by forecasting the total firm's systematic risk first, and then using the independent leverage estimate as an adjustment.

IV. Tests of the MM vs. Traditional Theories of Corporation Finance

To determine if the difference, ${}_B\beta - {}_A\beta$, found in this study is indeed the correct effect of leverage, some confirmation of the MM theory (since it was assumed to be correct up to this point) from the systematic risk approach is needed. Since a direct test by this approach seems impossible, an indirect, inferential test is suggested.

The MM theory [9, 10] predicts that for firms in the same risk-class, the capitalization rate if all the firms were financed with only common equity, $E(R_A)$, would be the same—regardless of the actual amount of debt and preferred each individual firm had. This would imply, from (11a), that if $E(R_A)$ must be the same for all firms in a risk-class, so must ${}_A\beta$. And if these firms had different ratios of fixed commitment obligations to common equity, this difference in financial risk would cause their observed ${}_B\beta$s to be different.

The major competing theory of corporation finance is what is now known as the "traditional theory," which has contrary implications. This theory predicts that the capitalization rate for common equity, $E(R_B)$, (sometimes called the required or expected stock yield, or expected earnings-price ratio) is constant, as debt is increased, up to some critical leverage point (this point being a function of gambler's ruin and bankruptcy costs).[8] The clear implication of this constant, horizontal, equity yield (or their initial downward sloping cost of capital curve) is that changes in market or covariability risk are assumed not to be discernible to the shareholders as debt is increased. Then the traditional theory is saying that the ${}_B\beta$s, a measure of this covariability risk, would be the same for all firms in a given risk-class irregardless of differences in leverage, as long as the critical leverage point is not reached.

Since there will always be unavoidable errors in estimating the β's of indi-

7. A faint, but possible, empirical indication of this point may be obtained from Table 1. The ratio of the mean point estimate to the mean standard error of estimate is less for the firm β than for the stock β in both the discrete and continuously compounded cases.

8. This interpretation of the traditional theory can be found in [9, especially their figure 2, page 275, and their equation (13) and footnote 24 where reference is made to Durand and Graham and Dodd].

TABLE 3
INDUSTRY MARKET VALUE RATIOS OF PREFERRED STOCK (P) AND DEBT (D) TO COMMON STOCK (S)

Industry Number	Industry	Number of Firms		P/S		D/S		$\frac{P+D}{S}$	
20	Food and Kindred Products	30	Mean*	0.22		0.81		1.04	
			ROM**	0.00	1.18	0.00	3.55	0.00	4.13
			ROCR***	0.00	2.52	0.00	8.10	0.00	10.01
28	Chemicals and Allied Products	30	Mean	0.07		0.25		0.33	
			ROM	0.00	0.51	0.00	0.90	0.00	1.20
			ROCR	0.00	1.54	0.00	2.07	0.00	2.92
29	Petroleum and Coal Products	18	Mean	0.06		0.22		0.27	
			ROM	0.00	0.26	0.00	0.55	0.03	0.57
			ROCR	0.00	0.83	0.00	1.54	0.00	2.30
33	Primary Metals	21	Mean	0.14		0.54		0.68	
			ROM	0.00	1.31	0.00	1.95	0.00	3.04
			ROCR	0.00	4.69	0.00	6.20	0.00	7.49
35	Machinery, except Electrical	28	Mean	0.07		0.33		0.40	
			ROM	0.00	0.49	0.00	1.92	0.00	2.32
			ROCR	0.00	1.28	0.00	6.92	0.00	7.62

TABLE 3 (*Continued*)

Industry Number	Industry	Number of Firms		P/S		D/S		$\frac{P+D}{S}$	
36	Electrical Machinery & Equipment	13	Mean	0.06		0.35		0.41	
			ROM	0.00	0.29	0.00	1.31	0.01	1.33
			ROCR	0.00	1.13	0.00	2.53	0.00	2.53
37	Transportation Equipment	24	Mean	0.08		0.38		0.47	
			ROM	0.00	0.54	0.00	0.93	0.00	1.32
			ROCR	0.00	2.33	0.00	3.76	0.00	6.09
49	Utilities	27	Mean	0.25		1.03		1.28	
			ROM	0.00	0.53	0.49	2.64	0.52	3.12
			ROCR	0.00	3.12	0.12	16.40	0.12	19.52
53	Dep't Stores, Order Houses & Vending Mach. Operators	17	Mean	0.13		0.49		0.62	
			ROM	0.00	0.38	0.01	1.52	0.01	1.87
			ROCR	0.00	1.09	0.00	3.19	0.00	3.66

* "Mean" refers to the average ratio over 20 years and over all firms in the industry.
** "Range of Means" (ROM) refers to the lowest firm's mean (over 20 years) ratio and the highest firm's mean (over 20 years) ratio in the industry.
*** "Range of Company Ranges" (ROCR) refers to the lowest and highest ratio in the industry, regardless of the year.

vidual firms and in specifying a risk-class, we would not expect to find a set of firms with identical systematic risk. But by specifying reasonable a priori risk-classes, if the individual firms had closer or less scattered $_A\beta$s than $_B\beta$s, then this would support the MM theory and contradict the traditional theory. If, instead, the $_B\beta$s were not discernibly more diverse than the $_A\beta$s, and the leverage ratio differed considerably among firms, then this would indicate support for the traditional theory.[9]

In order to test this implication, risk-classes must be first specified. The SEC two-digit industry classification was used for this purpose. Requiring enough firms for statistical reasons in any given industry, nine risk-classes were specified that had at least 13 firms; these nine classes are listed in Table 3 with their various leverage ratios.[10] It is clear from this table that our first requirement is met—that there is a considerable range of leverage ratios among firms in a risk-class and also over the twenty-year period.

Three tests will be performed to distinguish between the MM and traditional theories. The first is simply to calculate the standard deviation of the unbiased β estimates in a risk-class. The second is a chi-square test of the distribution of β's in an industry compared to the distribution of the β's in the total sample. Finally, an analysis of variance test on the estimated variance of the β's between industries, as opposed to within industries, is performed. In all tests, only the point estimate of β (which should be unbiased) for each stock and firm is used.[11]

The first test is reported in Table 4. If we compare the standard deviation of $_{AC}\beta$ with the standard deviation of $_{BC}\beta$ by industries (or risk-classes), we can note that $\sigma(_{AC}\beta)$ is less than $\sigma(_{BC}\beta)$ for eight out of the nine classes. The probability of obtaining this is only 0.0195, given a 50% probability that $\sigma(_{AC}\beta)$ can be larger or smaller than $\sigma(_{BC}\beta)$. These results indicate that the systematic risk of the firms in a given risk-class, if they were all financed only with common equity, is much less diverse than their observed stock's systematic risk. This supports the MM theory, at least in contrast to the traditional theory.[12]

9. The traditional theory also implies that $E(R_A)$ is equal to $E(R_B)$ for all firms. Unfortunately, we do not have a functional relationship between these traditional theory capitalization rates and the measured βs of this study. Clearly, since the $_A\beta$s were obtained assuming the validity of the MM theory, they would not be applicable for the traditional theory. In fact, no relationship between the $_A\beta$ and $_B\beta$ for a given firm, or for firms in a given risk-class, can be specified as was done for the capitalization rates.

10. The tenth largest industry had only eight firms. For our purpose of testing the uniformity of firm βs relative to stock βs within a risk-class, the use of the two-digit industry classification as a proxy does not seem as critical as, for instance, its use for the purpose of performing an MM valuation model study [8] wherein the ρ^τ must be pre-specified to be exactly the same for all firms in the industry.

11. Since these βs are estimated in the market model regressions with error, precise testing should incorporate the errors in the β estimation. Unfortunately, to do this is extremely difficult and more importantly, requires the normality assumption for the market model disturbance term. Since there is considerable evidence that is contrary to this required assumption [see 3], our tests will ignore the β measurement error entirely. But ignoring this is partially corrected in our first and third tests since means and variances of these point estimate βs must be calculated, and this procedure will "average out" the individual measurement errors by the factor 1/N.

12. Of course, there could always be another theory, as yet not formulated, which could be even

TABLE 4
MEAN AND STANDARD DEVIATION OF INDUSTRY β'S

Industry Number	Industry	Number of Firms		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
20	Food & Kindred Products	30	Mean β σ(β)	0.515 0.232	0.815 0.448	0.528 0.227	0.806 0.424
28	Chemicals & Allied Products	30	Mean β σ(β)	0.747 0.237	0.928 0.391	0.785 0.216	0.946 0.329
29	Petroleum & Coal Products	18	Mean β σ(β)	0.633 0.144	0.747 0.188	0.656 0.148	0.756 0.176
33	Primary Metals	21	Mean β σ(β)	1.036 0.223	1.399 0.272	1.106 0.197	1.436 0.268
35	Machinery, except Electrical	28	Mean β σ(β)	0.878 0.262	1.037 0.240	0.917 0.271	1.068 0.259
36	Electrical Machinery and Equipment	13	Mean β σ(β)	0.940 0.320	1.234 0.505	0.951 0.283	1.164 0.363
37	Transportation Equipment	24	Mean β σ(β)	0.860 0.225	1.062 0.313	0.875 0.225	1.048 0.289
49	Utilities	27	Mean β σ(β)	0.160 0.086	0.255 0.133	0.166 0.098	0.254 0.147
53	Department Stores, etc.	17	Mean β σ(β)	0.652 0.187	0.901 0.282	0.692 0.198	0.923 0.279

Our second test, the chi-square test, requires us to rank our 300 $_A\beta$s into ten equal categories, each with 30 $_A\beta$s (four miscellaneous firms were taken out randomly). By noting the value of the highest and lowest $_A\beta$ for each of the ten categories, a distribution of the number of $_A\beta$s in each category, by risk-class, can be obtained. This was then repeated for the other three betas. To test whether the distribution for each of the four β's and for each of the risk-classes follows the expected uniform distribution, a chi-square test was performed.[13]

Even with just casual inspection of these distributions of the betas by risk-class, it is clear that two industries, primary metals and utilities, are so highly skewed that they greatly exaggerate our results.[14] Eliminating these

more strongly supported than the MM theory. If we compare $\sigma(_A\beta)$ to $\sigma(_B\beta)$ by risk-classes in Table 4, precisely the same results are obtained as those reported above for the continuously-compounded betas.

13. By risk-classes, seven of the nine chi-square values of $_A\beta$ are larger than those of $_B\beta$, as are eight out of nine for the continuously-compounded betas. This would occur by chance with probabilities of 0.0898 and 0.0195, respectively, if there were a 50% chance that either the firm or stock chi-square value could be larger. Nevertheless, if we inspect the individual chi-square values by risk-class, we note that most of them are large so that the probabilities of obtaining these values are highly unlikely. For all four βs, the distributions for most of the risk-classes are nonuniform.

14. Primary metals have extremely large betas; utilities have extremely small betas.

two industries, and also two miscellaneous firms so that an even 250 firms are in the sample, new upper and lower values of the β's were obtained for each of the ten class intervals and for each of the four β's.

In Table 5, the chi-square values are presented; for the total of all risk-classes, the probability of obtaining a chi-square value less than 120.63 is over 99.95% (for $_A\beta$), whereas the probability of obtaining a chi-square value less than 99.75 is between 99.5% and 99.9% (for $_B\beta$). More sharply contrasting results are obtained when $_{AC}\beta$ is compared to $_{BC}\beta$. For $_{AC}\beta$, the probability of obtaining less than 128.47 is over 99.95%, whereas for $_{BC}\beta$, the probability of obtaining less than 78.65 is only 90.0%. By abstracting from financial risk, the underlying systematic risk is much less scattered when grouped into risk-classes than when leverage is assumed not to affect the systematic risk. The null hypothesis that the β's in a risk-class come from the same distribution as all β's is rejected for $_{AC}\beta$, but not for $_{BC}\beta$ (at the 90% level). Although this, in itself, does not tell us *how* a risk-class differs from the total market, an inspection of the distributions of the betas by risk-class underlying Table 5 does indicate more clustering of the $_{AC}\beta$s than the $_{BC}\beta$s so that the MM theory is again favored over the traditional theory.

The analysis of variance test is our last comparison of the implications of the two theories. The ratio of the estimated variance between industries to the estimated variance within the industries (the F-statistic) when the seven

TABLE 5
CHI-SQUARE RESULTS FOR ALL β'S AND ALL INDUSTRIES
(EXCEPT UTILITIES AND PRIMARY METALS)

Industry		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
Food and Kindred	Chi-Square	18.67	11.33	26.00	9.33
	$P\{\chi^2 < \}^* =$	95-97.5%	70-75%	99.5-99.9%	50-60%
Chemicals	Chi-Square	9.33	10.67	12.00	7.33
	$P\{\chi^2 < \} =$	50-60%	60-70%	75-80%	30-40%
Petroleum	Chi-Square	17.56	25.33	18.67	22.00
	$P\{\chi^2 < \} =$	95-97.5%	99.5-99.9%	95-97.5%	99-99.5%
Machinery	Chi-Square	19.14	12.00	24.86	9.14
	$P\{\chi^2 < \} =$	97.5-98%	75-80%	99.5-99.9%	50-60%
Electrical Machinery	Chi-Square	13.92	7.77	12.38	9.31
	$P\{\chi^2 < \} =$	80-90%	40-50%	80-90%	50-60%
Transportation Equipment	Chi-Square	15.17	16.83	13.50	6.83
	$P\{\chi^2 < \} =$	90-95%	90-95%	80-90%	30-40%
Dep't Stores	Chi-Square	14.18	3.59	14.18	3.59
	$P\{\chi^2 < \} =$	80-90%	5-10%	80-90%	5-10%
Miscellaneous	Chi-Square	12.67	12.22	6.89	11.11
	$P\{\chi^2 < \} =$	80-90%	80-90%	30-40%	70-75%
Total	Chi-Square	120.63	99.75	128.47	78.65
	$P\{\chi^2 < \} =$	over 99.95%	99.5-99.90%	over 99.95%	90.0%

* Example: $P\{\chi^2 < 18.67\} =$ 95-97.5% for 9 degrees of freedom.

industries are considered (again, the two obviously skewed industries, primary metals and utilities, were eliminated) is less for ${}_{B}\beta$ (F = 3.90) than for ${}_{A}\beta$ (F = 9.99), and less for ${}_{BC}\beta$ (F = 4.18) than for ${}_{AC}\beta$ (F = 10.83). The probability of obtaining these F-statistics for ${}_{A}\beta$ and ${}_{AC}\beta$ is less than 0.001, but for ${}_{B}\beta$ and ${}_{BC}\beta$ greater than or equal to 0.001. These results are consistent with the results obtained from our two previous tests. The MM theory is more compatible with the data than the traditional theory.[15]

V. Conclusions

This study attempted to tie together some of the notions associated with the field of corporation finance with those associated with security and portfolio analyses. Specifically, if the MM corporate tax leverage propositions are correct, then approximately 21 to 24% of the observed systematic risk of common stocks (when averaged over 304 firms) can be explained merely by the added financial risk taken on by the underlying firm with its use of debt and preferred stock. Corporate leverage does count considerably.

To determine whether the MM theory is correct, a number of tests on a contrasting implication of the MM and "traditional" theories of corporation finance were performed. The data confirmed MM's position, at least vis-à-vis our interpretation of the traditional theory's position. This should provide another piece of evidence on this controversial topic.

Finally, if the MM theory and the capital asset pricing model are correct, and if the adjustments made in equations (8) or (4a) result in accurate measures of the systematic risk of a leverage-free firm, the possibility is greater, without resorting to a fullblown risk-class study of the type MM did for the electric utility industry [8], of estimating the cost of capital for individual firms.

REFERENCES

1. M. Blume. "Portfolio Theory: A Step Toward Its Practical Application," *Journal of Business* 43 (April, 1970), 152-173.
2. P. Brown. "Some Aspects of Valuation in the Railroad Industry." Unpublished Ph.D. dissertation, Graduate School of Business, University of Chicago, 1968.
3. E. Fama. "The Behavior of Stock Market Prices," *Journal of Business* 38 (January, 1965), 34-105.
4. E. Fama, and M. Miller. *The Theory of Finance.* Chapter 4, Holt, Rinehart and Winston, 1972.
5. R. Hamada. "Portfolio Analysis, Market Equilibrium and Corporation Finance," *Journal of Finance* (March, 1969), 13-31.
6. J. Lintner. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (February, 1965), 13-37.
7. H. Markowitz. *Portfolio Selection: Efficient Diversification of Investments.* New York: John Wiley & Sons, Inc., 1959.
8. M. Miller, and F. Modigliani. "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-57," *American Economic Review* (June, 1966), 333-91.

15. All of our tests, it should be emphasized, although consistent, are only inferential. Aside from assuming that the two-digit SEC industry classification is a good proxy for risk-classes and that the errors in estimating the individual βs can be safely ignored, the tests rely on the two theories exhausting all the reasonable theories on leverage. But there is always the use of another line of reasoning. If the results of the MM electric utility study [8] are correct, and if these results can be generalized to all firms and to all risk-classes, then it can be claimed that the MM theory is universally valid. Then our result in Section III does indicate the correct effect of the firm's capital structure on the systematic risk of common stocks.

9. F. Modigliani, and M. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review* (June, 1958), 261–97.
10. ——————. "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review* (June, 1963), 433-43.
11. J. Mossin. "Equilibrium in a Capital Asset Market," *Econometrica* (October, 1966), 768-83.
12. W. Sharpe. "A Simplified Model for Portfolio Analysis," *Management Science* (January, 1963), 277-93.
13. ——————. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* (September, 1964), 425-42.

ohn Wiley & Sons is the oldest independent publishing company in With offices in North America, Europe, Australia, and Asia, Wiley ted to developing and marketing print and electronic products and stomers' professional and personal knowledge and understanding. ıance series contains books written specifically for finance and sionals as well as sophisticated individual investors and their Book topics range from portfolio management to e-commerce, risk ncial engineering, valuation, and financial instrument analysis, as '.

ailable titles, please visit our Web site at www.WileyFinance.com.

VALUATION

MEASURING AND MANAGING THE VALUE OF COMPANIES

FOURTH EDITION

McKinsey & Company

Tim Koller
Marc Goedhart
David Wessels

WILEY

JOHN WILEY & SONS, INC.

5

Frameworks for Valuation

In Part One, we built a conceptual framework to show what drives value. In particular, a company's value is driven, first, by its ability to earn a return on invested capital (ROIC) greater than its weighted average cost of capital (WACC), and second, by its ability to grow. High returns and growth result in high cash flows, which in turn drives value.

Part Two offers a step-by-step guide for analyzing and valuing a company in practice, including technical details for properly measuring and interpreting the drivers of value. This chapter provides a high-level summary of valuation models based on discounted cash flow (DCF). We show how these models lead to *identical* results when applied correctly, and we illustrate how they differ in their ease of implementation.

Among the many ways to value a company (see Exhibit 5.1 on page 102 for an overview), we focus on two: enterprise DCF and discounted economic profit. When applied correctly, both valuation methods yield the same results; however, each model has certain benefits in practice. Enterprise DCF remains the favorite of many practitioners and academics because it relies solely on the flow of cash in and out of the company, rather than on accounting-based earnings (which can be misleading). Discounted economic profit is gaining in popularity because of its close link to economic theory and competitive strategy. Economic profit highlights whether a company is earning its cost of capital in a given year. Given the methods' identical results and complementary benefits of interpretation, we use both enterprise DCF and economic profit when valuing a company.

Both the enterprise DCF and economic profit models discount future streams at the weighted average cost of capital. WACC-based models work best when a company maintains a relatively stable debt-to-value ratio. If a company's debt-to-value mix is expected to change, WACC-based models can

Exhibit 5.1 Frameworks for DCF-Based Valuation

Model	Measure	Discount factor	Assessment
Enterprise discounted cash flow	Free cash flow	Weighted average cost of capital	Works best for projects, business units, and companies that manage their capital structure to a target level.
Economic profit	Economic profit	Weighted average cost of capital	Explicitly highlights when a company creates value.
Adjusted present value	Free cash flow	Unlevered cost of equity	Highlights changing capital structure more easily than WACC-based models.
Capital cash flow	Capital cash flow	Unlevered cost of equity	Compresses free cash flow and the interest tax shield in one number, making it difficult to compare performance among companies and over time.
Equity cash flow	Cash flow to equity	Levered cost of equity	Difficult to implement correctly because capital structure is embedded within cash flow. Best used when valuing financial institutions.

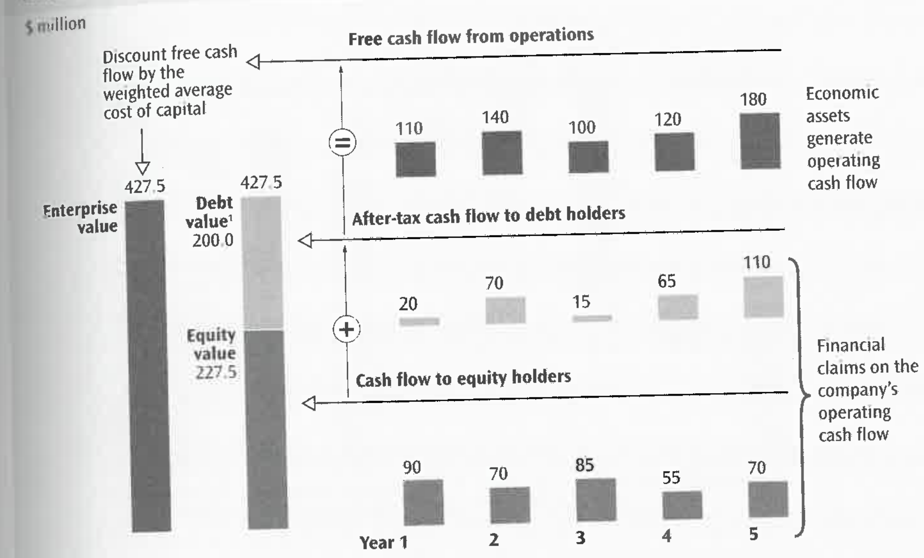
still yield accurate results but are more difficult to apply. When the company's capital structure is expected to change significantly, we recommend an alternative: adjusted present value (APV). Unlike WACC-based models, APV values the cash flow associated with capital structure (e.g., tax shields) separately from the cost of capital.

We conclude the chapter with a discussion of capital cash flow and equity cash flow valuation models. Because these two valuation models commingle operating performance and capital structure in cash flow, they lead more easily to mistakes in implementation. For this reason, we avoid capital cash flow and equity cash flow valuation models, except when valuing financial institutions, where capital structure is considered part of operations (for how to value financial institutions, see Chapter 25).

ENTERPRISE DISCOUNTED CASH FLOW MODEL

Enterprise valuation models value the company's operating cash flows. Equity valuation models, in contrast, value only the equity holder's claim against operating cash flows. In the 1950s, two Nobel laureates, Franco Modigliani and Merton Miller, postulated that the value of a company's economic assets must equal the value of the claims against those assets.

Exhibit 5.2 Enterprise Valuation of a Single-Business Company



[1] Debt value equals discounted after-tax cash flow to debt holders plus present value of interest tax shield.

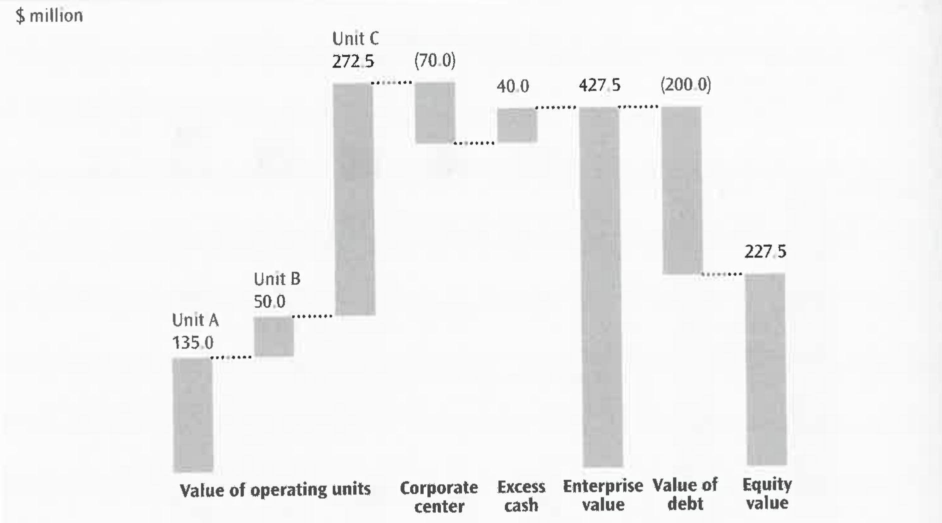
Thus, if we want to value the equity (and shares) of a company, we have two choices. We can value the company's operations and subtract the value of all nonequity financial claims (e.g., debt), or we can value the equity cash flows directly. In Exhibit 5.2, we demonstrate the relation between enterprise value and equity value. For this single-business company, equity can be calculated either directly at $227.5 million or by estimating enterprise value ($427.5 million) and subtracting debt ($200.0 million).

Although both methods lead to identical results when applied correctly, the equity method is difficult to implement in practice; matching equity cash flows with the correct cost of equity is challenging (for more on this, see the section on equity valuation later in this chapter). Consequently, to value a company's equity, we recommend valuing the enterprise first and then subtracting the value of any nonequity financial claims.[1]

In addition, the enterprise method is especially valuable when extended to a multibusiness company. As shown in Exhibit 5.3 on page 104, the enterprise value equals the summed value of the individual operating

[1] For financial institutions, such as banks and insurance companies, the choice, size, and structure of financial claims are directly linked to the company's operations (and thus are difficult to separate). In these situations, we prefer the equity cash-flow method. The valuation of financial institutions is addressed in Chapter 25.

Exhibit 5.3 Enterprise Valuation of a Multibusiness Company



units less the present value of the corporate center costs, plus the value of nonoperating assets. Using enterprise discounted cash flow, instead of the equity cash flow model, enables you to value individual projects, business units, and even the entire company with a consistent methodology.

To value a company's common stock using enterprise DCF:

1. Value the company's operations by discounting free cash flow from operations at the weighted average cost of capital.
2. Value nonoperating assets, such as excess marketable securities, nonconsolidated subsidiaries, and other equity investments. Combining the value of operating assets and nonoperating assets leads to enterprise value.
3. Identify and value all nonequity financial claims against the company's assets. Nonequity financial claims include (among others) fixed- and floating-rate debt, pension shortfalls, employee options, and preferred stock.
4. Subtract the value of nonequity financial claims from enterprise value to determine the value of common stock. To determine share price, divide equity value by the number of shares outstanding.

Exhibit 5.4 presents the results of an enterprise DCF valuation for Home Depot, the world's largest retailer of home improvement products.

Exhibit 5.4 Home Depot: Enterprise DCF Valuation

Year	Free cash flow (FCF) ($ million)	Discount factor (@ 9.3%)	Present value of FCF ($ million)
2004	1,930	0.915	1,766
2005	2,219	0.837	1,857
2006	2,539	0.766	1,944
2007	2,893	0.700	2,026
2008	3,283	0.641	2,104
2009	3,711	0.586	2,175
2010	4,180	0.536	2,241
2011	4,691	0.491	2,301
2012	5,246	0.449	2,355
2013	5,849	0.411	2,402
Continuing value	133,360	0.411	54,757
Present value of cash flow			75,928
Mid-year adjustment factor			1.046
Value of operations			**79,384**
Value of excess cash			1,609
Value of other nonoperating assets			84
Enterprise value			**81,077**
Value of debt			(1,365)
Value of capitalized operating leases			(6,554)
Equity value			**73,158**
Number of shares (at fiscal year-end 2003, million)			2,257
Estimated share value (in dollars)			**32.41**

To value Home Depot, future free cash flow is discounted to today's value and then summed across years. For simplicity, the first year's cash flow is discounted by one full year, the second by two full years, and so on. Since cash flows are generated throughout the year, and not as a lump sum, discounting in full-year increments understates the appropriate discount factor. Therefore, we adjust the present value by half a year,[2] leading to the value of operations of $79.4 billion.

To this value, add nonoperating assets (e.g., excess cash and other long-term nonoperating assets) to estimate Home Depot's enterprise value ($81.1 billion). From enterprise value, subtract the present value of nonequity

[2] A half-year adjustment is made to the present value for Home Depot because we assume cash flow is generated symmetrically around the midyear point. For companies dependent on year-end holidays, cash flows will be more heavily weighted toward the latter half of the year. In this case the adjustment should be smaller.

claims (traditional debt and capitalized operating leases) to arrive at Home Depot's estimated equity value ($73.2 billion). Dividing the equity value by the number of shares outstanding (2.3 billion) leads to an estimate of share value of $32.41. During the first half of 2004, Home Depot's stock price traded in the mid 30s.

Valuing Operations

The value of operations equals the discounted value of future free cash flow. Free cash flow equals the cash flow generated by the company's operations, less any reinvestment back into the business. Free cash flow is the cash flow available to *all* investors, and is *independent* of leverage. Consistent with this definition, free cash flow must be discounted using the weighted average cost of capital. The WACC is the company's opportunity cost of funds and represents a blended required return by the company's debt and equity holders.

Over the next few pages, we outline the enterprise DCF valuation process. Although we present it sequentially, valuation is an iterative process. To value operations, we analyze the company's historical performance; define and project free cash flow over the short, medium, and long run; and discount the projected free cash flows at the weighted average cost of capital.

Analyzing historical performance Before projecting future cash flow, examine the company's historical financial performance. A good analysis will focus on the key drivers of value: return on invested capital, growth, and free cash flow. By thoroughly analyzing the past, we can document whether the company has created value, whether it has grown, and how it compares with its competitors.

Although ROIC and FCF are critical to the valuation process, they cannot be computed directly from a company's reported financial statements. Whereas ROIC and FCF are intended to measure the company's operating performance, financial statements mix operating performance, nonoperating performance, and capital structure. Therefore, to calculate ROIC and FCF, first reorganize the accountant's financial statements into new statements that separate operating items, nonoperating items, and financial structure.

This reorganization leads to two new terms: invested capital and net operating profits less adjusted taxes (NOPLAT). Invested capital represents the investor capital required to fund operations, without distinguishing how the capital is financed. NOPLAT represents the total after-tax operating income generated by the company's invested capital, available to *all* financial investors.

Exhibit 5.5 presents the historical NOPLAT and invested capital for Home Depot and one of its direct competitors, Lowe's. To calculate ROIC, divide NOPLAT by average invested capital. In 2003, Home Depot's return on invested capital equaled 18.2 percent (based on a two-year average of invested capital), which exceeds its weighted average cost of capital of 9.3 percent. A detailed discussion of invested capital and NOPLAT, as well as an in-depth historical examination of Home Depot and Lowe's, is presented in Chapter 7.

Next, use the reorganized financial statements to calculate free cash flow, which will be the basis for our valuation. Defined in a manner consistent with ROIC, free cash flow relies on NOPLAT and the change in invested capital. Unlike the accountant's cash flow statement (provided in the company's annual report), free cash flow is independent of nonoperating items and capital structure.

Exhibit 5.6 on page 108 presents historical free cash flow for both Home Depot and Lowe's. As seen in the exhibit, Home Depot is generating nearly

Exhibit 5.5 Home Depot & Lowe's: Historical ROIC Analysis

$ million

	Home Depot			Lowe's		
	2001	2002	2003	2001	2002	2003
Net sales	53,553	58,247	64,816	22,111	26,491	30,838
Cost of merchandise sold	(37,406)	(40,139)	(44,236)	(15,743)	(18,465)	(21,231)
Selling, general and administrative	(10,451)	(11,375)	(12,658)	(4,053)	(4,859)	(5,671)
Depreciation	(756)	(895)	(1,075)	(517)	(626)	(758)
Operating lease interest	288	260	276	106	106	114
Adjusted EBITA	5,228	6,098	7,123	1,904	2,647	3,292
Adjusted taxes	(2,020)	(2,117)	(2,040)	(654)	(825)	(1,069)
NOPLAT	**3,208**	**3,981**	**5,083**	**1,250**	**1,822**	**2,223**
Invested capital						
Operating working capital	2,552	2,746	2,674	1,634	1,451	1,363
Net property and equipment	15,375	17,168	20,063	8,653	10,352	11,945
Capitalized operating leases	5,459	5,890	6,554	2,189	2,373	2,762
Net other assets	(216)	(247)	(524)	134	145	211
Invested capital (excluding goodwill)	23,170	25,557	28,767	12,611	14,321	16,281
Acquired intangibles and goodwill	419	575	833	0	0	0
Cumulative amortization and unreported goodwill	46	54	55	730	730	730
Invested capital (including goodwill)	**23,635**	**26,185**	**29,655**	**13,341**	**15,051**	**17,012**
ROIC excluding goodwill (average)	14.5%	16.3%	18.7%	10.9%	13.5%	14.5%
ROIC including goodwill (average)	14.3%	16.0%	18.2%	10.3%	12.8%	13.9%

Exhibit 5.6 Home Depot and Lowe's: Historical Free Cash Flow

$ million

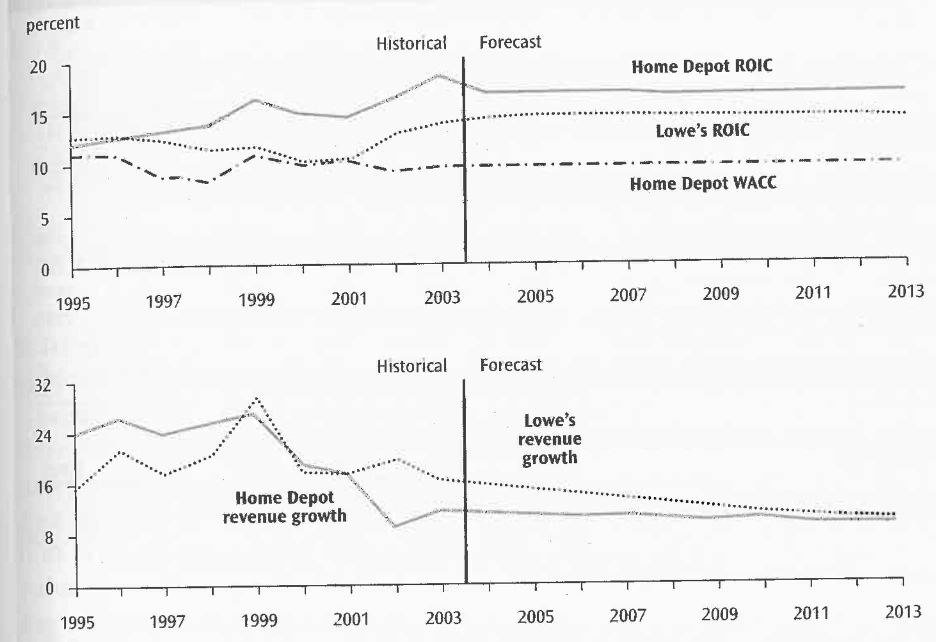
	Home Depot			Lowe's		
	2001	2002	2003	2001	2002	2003
NOPLAT	3,208	3,981	5,083	1,250	1,822	2,223
Depreciation	756	895	1,075	517	626	758
Gross cash flow	3,964	4,876	6,157	1,767	2,448	2,981
Investment in operating working capital	834	(194)	72	(203)	183	88
Net capital expenditures	(3,063)	(2,688)	(3,970)	(2,135)	(2,325)	(2,351)
Investment in capitalized operating leases	(775)	(430)	(664)	(547)	(184)	(389)
Investments in intangibles and goodwill	(113)	(164)	(259)	0	0	0
Decrease (increase) in other operating assets	105	31	277	(7)	(11)	(66)
Increase (decrease) in accumulated other comprehensive income	(153)	138	172	3	0	0
Gross investment	(3,165)	(3,307)	(4,372)	(2,889)	(2,336)	(2,719)
Free cash flow	**799**	**1,569**	**1,785**	**1,122**	**112**	**262**
After-tax interest income	33	49	36	15	13	9
Decrease (increase) in excess cash	(1,509)	383	(473)	(321)	(189)	(415)
Decrease (increase) in nonoperating assets	9	(24)	23	13	(7)	(140)
Discontinued operations	0	0	0	0	0	15
Cash flow available to investors	**(668)**	**1,977**	**1,371**	**(1,415)**	**(71)**	**(268)**

	2001	2002	2003	2001	2002	2003
After-tax interest expense	17	23	38	123	125	121
After-tax lease interest expense	177	162	170	66	65	71
Decrease (increase) in debt	88	140	(44)	(903)	78	60
Decrease (increase) in capitalized operating leases	(775)	(430)	(664)	(547)	(184)	(389)
Flows to debt holders	(492)	(105)	(500)	(1,261)	85	(138)
Dividends	396	492	595	60	66	87
Net shares repurchased (issued)	(572)	1,590	1,276	(213)	(222)	(217)
Flows to equity holders	(176)	2,082	1,871	(154)	(156)	(130)
Cash flow available to investors	**(668)**	**1,977**	**1,371**	**(1,415)**	**(71)**	**(268)**

$2 billion in free cash flow, whereas Lowe's free cash flow is barely positive. This isn't necessarily a problem for Lowe's. The company's free cash flow is small because it is reinvesting most of its gross cash flow to grow its business.

Projecting revenue growth, ROIC, and free cash flow To build an enterprise DCF valuation, we project revenue growth, return on invested capital, and free cash flow. Exhibit 5.7 graphs historical and projected ROIC and

Exhibit 5.7 ROIC and Revenue Growth Projections



revenue growth for Home Depot and Lowe's. As the graphs demonstrate, the two companies are transitioning from a period of high growth (25 percent annually) into mature businesses with strong ROICs (well above Home Depot's 9.3 percent cost of capital) and lower growth rates (currently 10 to 15 percent but falling to 5 percent over the next 10 years).

Free cash flow, which is driven by revenue growth and ROIC, provides the basis for enterprise DCF valuation. Exhibit 5.8 on page 110 shows a summarized free cash flow calculation for Home Depot.[3] To forecast Home Depot's free cash flow, start with forecasts of NOPLAT and invested capital. Over the short run (the first few years), forecast all financial statement line items, such as gross margin, selling expenses, accounts receivable, and inventory. Moving farther out, individual line items become difficult to project. Therefore, over the medium horizon (5 to 10 years), focus on the company's key value drivers, such as operating margin, adjusted tax rate, and capital efficiency. At some point, even projecting key drivers on a

[3] Free cash flow does not incorporate any financing-related cash flows such as interest expense or dividends. A good stress test for an enterprise valuation model is to change future interest rates or dividend payout ratios and observe free cash flow. Free cash flow forecasts should *not* change when you adjust the cost of debt or dividend policy.

Exhibit 5.8 Home Depot: Free Cash Flow Summary

$ million

	Historical 2001	2002	2003	Forecast 2004	2005	2006
NOPLAT	3,208	3,981	5,083	5,185	5,741	6,342
Depreciation	756	895	1,075	1,193	1,321	1,459
Gross cash flow	3,964	4,876	6,157	6,378	7,062	7,801
Investment in operating working capital	834	(194)	72	(294)	(318)	(344)
Net capital expenditures	(3,063)	(2,688)	(3,970)	(3,399)	(3,708)	(4,036)
Investment in capitalized operating leases	(775)	(430)	(664)	(721)	(780)	(842)
Investments in intangibles and goodwill	(113)	(164)	(259)	(92)	(99)	(107)
Decrease (increase) in other operating assets	105	31	277	58	62	67
Increase (decrease) in accumulated other comprehensive income	(153)	138	172	0	0	0
Gross investment	(3,165)	(3,307)	(4,372)	(4,448)	(4,843)	(5,261)
Free cash flow	799	1,569	1,785	1,930	2,219	2,539

year-by-year basis becomes meaningless. To value cash flows beyond this point, use a continuing-value formula, described next.

Estimating continuing value At the point where predicting the individual key value drivers on a year-by-year basis becomes impractical, do not vary the individual drivers over time. Instead, use a perpetuity-based continuing value, such that:

$$\text{Value of Operations} = \frac{\text{Present Value of Free Cash Flow}}{\textit{during}\text{ Explicit Forecast Period}} + \frac{\text{Present Value of Free Cash Flow}}{\textit{after}\text{ Explicit Forecast Period}}$$

Although many continuing-value models exist, we prefer the key value driver model presented in Chapter 3. The key value driver formula is superior to alternative methodologies because it is based on cash flow and links cash flow to growth and ROIC. The key value driver formula is:

$$\text{Continuing Value}_t = \frac{\text{NOPLAT}_{t+1}\left(1 - \frac{g}{\text{RONIC}}\right)}{\text{WACC} - g}$$

The formula requires a forecast of net operating profits less adjusted taxes (NOPLAT) in the year following the explicit forecast period, the long-run

Exhibit 5.9 Home Depot: Continuing Value

$ million

NOPLAT $_{2014}$	12,415
Return on incremental invested capital (RONIC)	9.3%
NOPLAT growth rate in perpetuity (g)	4.0%
Weighted average cost of capital (WACC)	9.3%

$$\text{Continuing Value}_t = \frac{\text{NOPLAT}_{t+1}\left(1 - \frac{g}{\text{RONIC}}\right)}{\text{WACC} - g}$$

$$= 133{,}360$$

forecast for return on new capital (RONIC), the weighted average cost of capital (WACC), and long-run growth in NOPLAT (g).

Exhibit 5.9 presents an estimate for Home Depot's continuing value. Based on a final-year estimate of NOPLAT ($12.4 billion), return on new investment equal to the cost of capital (9.3 percent), and a long-term growth rate of 4 percent, the continuing value is estimated at $133.4 billion. This value is then discounted into today's dollars and added to the value from the explicit forecast period to determine Home Depot's operating value (see Exhibit 5.4).

Alternative methods and additional details for estimating continuing value are provided in Chapter 9.

Discounting free cash flow at the weighted average cost of capital To determine the value of operations, discount each year's forecast of free cash flow for time and risk. When you discount any set of cash flows, make sure to define the cash flows and discount factor consistently. Since free cash flows are available to *all* investors, the discount factor for free cash flow must represent the risk faced by all investors. The weighted average cost of capital (WACC) blends the required rates of return for debt (k_d) and equity (k_e) based on their market-based target values. For a company financed solely with debt and equity, the WACC is defined as follows:

$$\text{WACC} = \frac{D}{D+E}k_d(1-T_m) + \frac{E}{D+E}k_e$$

Note how the cost of debt has been reduced by the marginal tax rate (T_m). We do this because the interest tax shield has been *excluded* from free cash flow (remember, interest is tax deductible). Since the interest tax shield has value, it must be incorporated in the valuation. Enterprise DCF values the tax shield by reducing the weighted average cost of capital.

Why move the interest tax shields from free cash flow to the cost of capital? By calculating free cash flow as if the company were financed entirely

Exhibit 5.10 Home Depot: Weighted Average Cost of Capital

percent

Source of capital	Proportion of total capital	Cost of capital	Marginal tax rate	After-tax opportunity cost	Contribution to weighted average
Debt	8.3	4.7	38.2	2.9	0.2
Equity	91.7	9.9		9.9	9.1
WACC	100.0				9.3

with equity, we can compare operating performance across companies and over time without regard to capital structure. By focusing solely on operations, we can develop a cleaner picture of historical performance, and this leads to better forecasting.

Although applying the weighted average cost of capital is intuitive and relatively straightforward, it comes with some drawbacks. If you discount all future cash flows with a constant cost of capital, as most analysts do, you are implicitly assuming the company manages its capital structure to a target rate. For example, if a company plans to increase its debt-to-value ratio, the current cost of capital will understate the expected tax shields. Although the WACC can be adjusted for a changing capital structure, the process is complicated. In these situations, we recommend an alternative method such as adjusted present value.

The weighted average cost of capital for Home Depot is presented in Exhibit 5.10. For simplicity, the cost of capital in this valuation is based on the company's current capital structure. Since Home Depot has very little debt, the weighted average cost of capital (9.3 percent) is very close to Home Depot's cost of equity (9.9 percent). Chapter 10 provides a more formal discussion of WACC and its components.

This cost of capital is used to discount each year's forecasted cash flow, as well as the continuing value. The result is the value of operations.

Identifying and Valuing Nonoperating Assets

When measured properly, free cash flow from operations should not include any cash flows from nonoperating assets. Instead, nonoperating assets should be valued separately. Nonoperating assets can be segmented into two groups, marketable securities and illiquid investments.

Excess cash and marketable securities Statement of Financial Accounting Standards (SFAS) No. 115 (1993) and International Accounting Standards

(IAS) No. 39 (1998) require companies to report liquid debt and equity investments (e.g., excess cash and marketable securities) at a fair market value on the company's balance sheet.[4] Therefore, when valuing *liquid* nonoperating assets, use their most recent reported balance sheet value, rather than discount future nonoperating flows.

Illiquid investments, such as nonconsolidated subsidiaries When valuing a company from the inside, you should value illiquid investments by using enterprise DCF (i.e., project cash flow and discount at the appropriate cost of capital). If you are valuing the company from the outside, valuation of these assets is rough at best. Companies disclose very little information about illiquid investments, such as discontinued operations, excess real estate, nonconsolidated subsidiaries, and other equity investments.

For nonconsolidated subsidiaries, information disclosure depends on the level of ownership. When a company has some influence but not a controlling interest[5] in another company, it records its portion of the subsidiary's profits on its own income statement and the original investment plus its portion of reinvested profits on its own balance sheet. Use this information to create a simple cash flow statement. To discount the cash flow, use a cost of capital commensurate with the risk of the investment, not the parent company's cost of capital (this is why we recommend separation of operating and nonoperating assets).

When ownership is less than 20 percent, investments are reported at historical cost, and the company's portion of profits is recorded *only* when paid out to the parent. In most situations, you will see nothing more than the investment's original cost. In this case, use a multiple of the book value or a tracking portfolio to value the investment. Further details for valuing nonoperating assets are covered in Chapter 11.

Identifying and Valuing Nonequity Claims

Add the value of nonoperating assets to the value of operations to determine enterprise value. To estimate equity value, subtract any nonequity claims, such as debt, unfunded retirement liabilities, capitalized operating leases, and outstanding employee options. Common equity is a residual claimant, receiving cash flows only after the company has fulfilled its other contractual claims. In today's increasingly complex financial markets, *many* claimants have rights to a company's cash flow before equity

[4] Liquid investments can appear as either current or long-term assets. Their placement depends on when management intends to sell the assets.

[5] In the United States and Europe, this is generally accepted as between 20 percent and 50 percent ownership.

holders—and they are not always easy to spot. Enron collapsed in 2001 under the weight of hidden debt. The company signed agreements with the creditors of its nonconsolidated subsidiaries, promising to cover loan payments if the subsidiaries could not.[6] Since the subsidiaries were not consolidated, the debt never appeared on Enron's balance sheet, and investors dramatically overestimated the equity's value. When the loans were disclosed in November 2001, the company's stock price fell by more than 50 percent in a single week.

Here are the most common nonequity claims:

- *Debt:* If available, use the market value of all outstanding debt, including fixed and floating rate debt. If that information is unavailable, the book value of debt is a reasonable proxy, unless the probability of default is high or interest rates have changed dramatically since the debt was originally issued. Any valuation of debt, however, should be consistent with your estimates of enterprise value. (See Chapter 11 for more details.)
- *Unfunded retirement liabilities:* The recent weak performance of global stock markets and the rising cost of health care have left many companies with retirement liabilities that are partially unfunded. Although the actual shortfall is not reported on the balance sheet (only a smoothed amount is transferred to the balance sheet), the stock market clearly values unfunded retirement liabilities as an offset against enterprise value. Consider General Motors, which raised nearly $20 billion in debt to fund its pension deficit. The company's stock price actually rose during the month when the new debt was announced and issued. Investors knew a liability existed, even though it wasn't on the balance sheet.
- *Operating leases:* These represent the most common form of off-balance-sheet debt. Under certain conditions, companies can avoid capitalizing leases as debt on their balance sheet, although required payments must be disclosed in the footnotes.
- *Contingent liabilities:* Any other material off-balance-sheet contingencies, such as lawsuits and loan guarantees, will be reported in the footnotes.
- *Preferred stock:* Although the name denotes equity, preferred stock in well-established companies more closely resembles unsecured debt. Therefore, preferred-stock dividends (which are often predetermined and required) should be valued separately, using an appropriate risk-adjusted discount rate.

[6] D. Henry, "Who Else Is Hiding Debt: Moving Financial Obligations into Off-Book Vehicles Is Now a Common Ploy," *BusinessWeek* (January 2002), p. 36.

- *Employee options:* Each year, many companies offer their employees compensation in the form of options. Since options give the employee the right to buy company stock at a potentially discounted price, they can have great value. Employee options can be valued using traditional models, such as Black-Scholes, or advanced techniques such as lattice models.
- *Minority interest:* When a company controls a subsidiary but does not own 100 percent, the investment must be consolidated on the parent company's balance sheet, and the funding other investors provide is recognized on the parent company's balance sheet as minority interest. When valuing minority interest, it is important to realize the minority interest holder does not have a claim on the company's assets, but rather a claim on the subsidiary's assets. Thus, minority interest must be valued separately and not as a percentage of company value.

The identification and valuation of nonequity financial claims are covered in detail in Chapter 11.

A common mistake made when valuing companies is to double-count claims already deducted from cash flow. Consider a company with a pension shortfall. You have been told the company will make extra payments to eliminate the liability. If you deduct the present value of the liability from enterprise value, you should not model the extra payments within free cash flow; that would mean double-counting the shortfall (once in cash flow and once as a claim), leading to an underestimate of equity value.

Valuing Equity

Once you have identified and valued all nonequity claims, we can subtract the claims from enterprise value to determine equity value. Home Depot has traditional debt ($1.4 billion) and capitalized operating leases ($6.6 billion). To value Home Depot's common stock, we subtract each of these claims from Home Depot's enterprise value (see Exhibit 5.4).

To determine Home Depot's share price, divide the estimated common-stock value by the number of undiluted shares outstanding. Do not use diluted shares. We have already valued convertible debt and employee stock options separately. If we were to use diluted shares, we would be double-counting the options' value.

At the end of fiscal year 2003, Home Depot had 2.3 billion shares outstanding. Dividing the equity estimate of $73.2 billion by 2.3 billion shares generates an estimated value of $32 per share. The estimated share value assumes Home Depot can maintain its current ROIC over the forecast period and the growth rate will remain strong, decaying gradually over the next 10 years from its current level of 11 percent to 4 percent in the continuing value. During the first half of 2004, the Home Depot's actual stock price traded between $32 and $38 per share.

ECONOMIC-PROFIT-BASED VALUATION MODELS

The enterprise DCF model is a favorite of academics and practitioners alike because it relies solely on how cash flows in and out of the company. Complex accounting can be replaced with a simple question: Does cash change hands? One shortfall of enterprise DCF, however, is that each year's cash flow provides little insight into the company's performance. Declining free cash flow can signal either poor performance or investment for the future. The economic profit model highlights how and when the company creates value yet leads to a valuation that is identical to that of enterprise DCF.

As stated in Chapter 3, economic profit measures the value created by the company in a single period and is defined as follows:

$$\text{Economic Profit} = \text{Invested Capital} \times (\text{ROIC} - \text{WACC})$$

Since ROIC equals NOPLAT divided by invested capital, we can rewrite the equation as follows:

$$\text{Economic Profit} = \text{NOPLAT} - (\text{Invested Capital} \times \text{WACC})$$

In Exhibit 5.11, we present economic profit calculations for Home Depot using both methods. Since Home Depot has been earning returns greater than its cost of capital, its historical economic profit is positive. Given the company's strong competitive position, we also project positive economic profits going forward. Not every company has positive economic profit. In fact, many companies earn an accounting profit (net income greater than zero) but do not earn their cost of capital.

To demonstrate how economic profit can be used to value a company—and to demonstrate its equivalence to enterprise DCF, consider a stream of growing cash flows valued using the growing-perpetuity formula:

$$\text{Value}_0 = \frac{\text{FCF}_1}{\text{WACC} - g}$$

In Chapter 3, we transformed this cash flow perpetuity into the key value driver model. The key value driver model is superior to the simple cash flow perpetuity model, because it explicitly models the relation between growth and required investment. Using a few additional algebraic

Exhibit 5.11 Home Depot: Economic Profit Summary

$ million

	Historical 2001	2002	2003	Forecast 2004	2005	2006
Method 1						
Return on invested capital	15.0%	16.8%	19.4%	17.5%	17.4%	17.4%
Weighted average cost of capital	10.1%	9.0%	9.3%	9.3%	9.3%	9.3%
Economic spread	4.9%	7.9%	10.1%	8.2%	8.1%	8.1%
Invested capital	21,379	23,635	26,185	29,655	32,910	36,432
Economic profit	**1,048**	**1,857**	**2,645**	**2,424**	**2,677**	**2,950**
Method 2						
Invested capital	21,379	23,635	26,185	29,655	32,910	36,432
Weighted average cost of capital	10.1%	9.0%	9.3%	9.3%	9.3%	9.3%
Capital charge	2,159	2,124	2,438	2,761	3,064	3,392
NOPLAT	3,208	3,981	5,083	5,185	5,741	6,342
Capital charge	2,159	2,124	2,438	2,761	3,064	3,392
Economic profit	**1,048**	**1,857**	**2,645**	**2,424**	**2,677**	**2,950**

steps (see Appendix A) and the assumption that the company's ROIC on new projects equals historical ROIC, we can transform the cash flow perpetuity into a key value driver model based on economic profits:

$$\text{Value}_0 = \text{Invested Capital}_0 + \frac{\text{Invested Capital}_0 \times (\text{ROIC} - \text{WACC})}{\text{WACC} - g}$$

Finally, we substitute the definition of economic profit:

$$\text{Value}_0 = \text{Invested Capital}_0 + \frac{\text{Economic Profit}_1}{\text{WACC} - g}$$

As can be seen in the economic-profit-based key value driver model, the operating value of a company equals its book value of invested capital plus the present value of all future value created. In this case, the future economic profits are valued using a growing perpetuity, because the company's economic profits are increasing at a constant rate over time. More generally, economic profit can be valued as follows:

$$\text{Value}_0 = \text{Invested Capital}_0 + \sum_{t=1}^{\infty} \frac{\text{Invested Capital}_{t-1} \times (\text{ROIC}_t - \text{WACC})}{(1 + \text{WACC})^t}$$

Since the economic profit valuation was derived directly from the free cash flow model (see Appendix B for a proof of equivalence), any valuation based on discounted economic profits will be identical to enterprise DCF. To assure equivalence, however, you must:

- Use beginning-of-year invested capital (i.e., last year's value).
- Use the same invested-capital number for both economic profit and ROIC. For example, ROIC can be measured either with or without goodwill. If you measure ROIC without goodwill, invested capital must also be measured without goodwill. All told, it doesn't matter how you define invested capital, as long as you are consistent.

Exhibit 5.12 presents the valuation results for Home Depot using economic profit. Economic profits are explicitly forecasted for 10 years; the

Exhibit 5.12 Home Depot: Economic Profit Valuation

Year	Invested capital[1] ($ million)	ROIC (percent)	WACC (percent)	Economic profit ($ million)	Discount factor (@ 9.3%)	Present value of economic profit ($ million)
2004	29,655	17.5	9.3	2,424	0.915	2,217
2005	32,910	17.4	9.3	2,677	0.837	2,241
2006	36,432	17.4	9.3	2,950	0.766	2,259
2007	40,235	17.4	9.3	3,242	0.700	2,271
2008	44,329	17.3	9.3	3,556	0.641	2,278
2009	48,729	17.3	9.3	3,890	0.586	2,281
2010	53,445	17.3	9.3	4,247	0.536	2,278
2011	58,488	17.2	9.3	4,627	0.491	2,270
2012	63,870	17.2	9.3	5,031	0.449	2,258
2013	69,600	17.2	9.3	5,458	0.411	2,241
Continuing value				57,671	0.411	23,679
Present value of economic profit						46,273
Invested capital[1] $_{2004}$						29,655
Invested capital plus present value of economic profit						75,928
Mid-year adjustment factor						1.046
Value of operations						**79,384**
Value of excess cash						1,609
Value of other nonoperating assets						84
Enterprise value						**81,077**
Value of debt						(1,365)
Value of capitalized operating leases						(6,554)
Equity value						**73,158**

[1]Invested capital is measured at the beginning of the year.

remaining years are valued using an economic profit continuing-value formula.[7] Comparing the equity value from Exhibit 5.4 with that of Exhibit 5.12, we see that the value of Home Depot's stock is the same, regardless of the method.

The benefit of economic profit becomes apparent when we examine the drivers of economic profit, ROIC and WACC, on a year-by-year basis in Exhibit 5.12. Notice how the valuation depends heavily on Home Depot's ability to maintain current levels of ROIC (17.5 percent) well above the WACC (9.3 percent). If the company's markets become saturated, growth could become elusive, and some companies might compete on price to steal market share. If this occurs, ROICs will drop, and economic profits will revert to zero. Explicitly modeling ROIC as a primary driver of economic profit will prominently display this analysis. Conversely, the free cash flow model fails to show this dynamic. Free cash flow could continue to grow, even as ROIC falls.

Another insight generated by the economic profit model occurs when comparing a company's value of operations with its invested capital. For Home Depot, the estimated operating value ($79.4 billion) exceeds the company's invested capital ($29.7 billion) by more than $49.7 billion.

ADJUSTED PRESENT VALUE MODEL

When building an enterprise DCF or economic profit valuation, most financial analysts discount all future flows at a *constant* weighted average cost of capital. Using a constant WACC, however, assumes the company manages its capital structure to a target debt-to-value ratio.

In most situations, debt grows in line with company value. But suppose the company planned to significantly change its capital structure. Indeed, companies with significant debt often pay it down as cash flow improves, thus lowering their future debt-to-value ratios. In these cases, a valuation

[7]To calculate continuing value, you can use the economic-profit-based key value driver formula, but only if RONIC equals historical ROIC in the continuing-value year. If RONIC going forward differs from the final year's ROIC, then the equation must be separated into current and future economic profits:

$$\text{Continuing Value}_t = \underbrace{\frac{IC_t\,(ROIC_{t+1} - WACC)}{WACC}}_{\text{Current Economic Profits}} + \underbrace{\frac{PV(\text{Economic Profit}_{t+2})}{WACC - g}}_{\text{Future Economic Profits}}$$

such that

$$PV(\text{Economic Profit}_{t+2}) = \frac{NOPLAT_{t+1}\left(\frac{g}{RONIC}\right)(RONIC - WACC)}{WACC}$$

based on a constant WACC would overstate the value of the tax shields. Although the WACC can be adjusted yearly to handle a changing capital structure, the process is complex. Therefore, we turn to an alternative model: adjusted present value.

The adjusted present value (APV) model separates the value of operations into two components: the value of operations as if the company were all-equity financed and the value of tax shields that arise from debt financing:[8]

$$\text{Adjusted Present Value} = \text{Enterprise Value as if the Company Was All-Equity Financed} + \text{Present Value of Tax Shields}$$

The APV valuation model follows directly from the teachings of Modigliani and Miller, who proposed that in a market with no taxes (among other things), a company's choice of financial structure will not affect the value of its economic assets. Only market imperfections, such as taxes and distress costs, affect enterprise value.

When building a valuation model, it is easy to forget these teachings. To see this, imagine a company (in a world with no taxes) that has a 50-50 mix of debt and equity. If the company's debt has an expected return of 5 percent and the company's equity has an expected return of 15 percent, its weighted average cost of capital would be 10 percent. Suppose the company decides to issue more debt, using the proceeds to repurchase shares. Since the cost of debt is lower than the cost of equity, it would appear that issuing debt to retire equity should lower the WACC, raising the company's value.

This line of thinking is flawed, however. In a world without taxes, a change in capital structure would *not* change the cash flow generated by operations, nor the risk of those cash flows. Therefore, neither the company's enterprise value nor its cost of capital would change. So why did we think it would? When adding debt, we adjusted the weights, but we failed to properly increase the cost of equity. Since debt payments have priority over cash flows to equity, adding leverage increases the risk to equity holders. When leverage rises, they demand a higher return. Modigliani and Miller postulated this increase would perfectly offset the change in weights.

In reality, taxes play a part in decision making, and capital structure choice therefore *can* affect cash flows. Since interest is tax deductible, profitable companies can lower taxes by raising debt. But, if the company relies

[8] In this book, we focus on the tax shields generated by interest. On a more general basis, the APV values *any* incremental cash flows associated with capital structure, such as tax shields, issue costs, and distress costs. Distress costs include direct costs, such as court-related fees, and indirect costs, such as the loss of customers and suppliers.

too heavily on debt, the company's customers and suppliers may fear bankruptcy and walk away, restricting future cash flow (academics call this distress costs or deadweight costs). Rather than model the effect of capital structure changes in the weighted average cost of capital, APV explicitly measures and values the cash flow effects of financing separately.

To build an APV-based valuation, value the company as if it were all-equity financed. Do this by discounting free cash flow by the unlevered cost of equity (what the cost of equity would be if the company had no debt). To this value, add any value created by the company's use of debt. Exhibit 5.13 values Home Depot using adjusted present value. Since we assume that Home Depot will manage its capital structure to a target debt-to-value level of 9.3 percent, the APV-based valuation leads to the same value for equity as

Exhibit 5.13 Home Depot: Valuation Using Adjusted Present Value

Year	Free cash flow ($ million)	Interest tax shield (ITS)	Discount factor (@ 9.5%)	Present value of FCF ($ million)	Present value of ITS ($ million)
2004	1,930	113	0.914	1,763	103
2005	2,219	120	0.835	1,852	100
2006	2,539	128	0.763	1,936	98
2007	2,893	136	0.697	2,016	95
2008	3,283	145	0.636	2,090	92
2009	3,711	153	0.581	2,158	89
2010	4,180	162	0.531	2,220	86
2011	4,691	171	0.485	2,276	83
2012	5,246	180	0.443	2,326	80
2013	5,849	189	0.405	2,369	77
Continuing value	129,734	3,626	0.405	52,550	1,469
Present value				73,557	2,372
Present value of FCF using unlevered cost of equity					73,557
Present value of interest tax shields (ITS)					2,372
Present value of FCF and ITS					75,928
Mid-year adjustment factor					1.046
Value of operations					**79,384**
Value of excess cash					1,609
Value of other nonoperating assets					84
Enterprise value					**81,077**
Value of debt					(1,365)
Value of capitalized operating leases					(6,554)
Equity value					**73,158**

did enterprise DCF (see Exhibit 5.4) and economic profit (see Exhibit 5.12). A simplified proof of equivalence between enterprise DCF and adjusted present value can be found in Appendix C. The following subsections explain APV in detail.

Value Free Cash Flow at Unlevered Cost of Equity

When valuing a company using the APV, we explicitly separate the unlevered value of operations (V_u) from any value created by financing, such as tax shields (V_{txa}). For a company with debt (D) and equity (E), this relation is as follows:

$$V_u + V_{txa} = D + E \tag{1}$$

A second result of Modigliani and Miller's work is that the total risk of the company's assets, real and financial, must equal the total risk of the financial claims against those assets. Thus, in equilibrium, the blended cost of capital for operating assets (k_u, which we call the unlevered cost of equity) and financial assets (k_{txa}) must equal the blended cost of capital for debt (k_d) and equity (k_e):

$$\underbrace{\frac{V_u}{V_u + V_{txa}} k_u}_{\text{Operating Assets}} + \underbrace{\frac{V_{txa}}{V_u + V_{txa}} k_{txa}}_{\text{Tax Assets}} = \underbrace{\frac{D}{D+E} k_d}_{\text{Debt}} + \underbrace{\frac{E}{D+E} k_e}_{\text{Equity}} \tag{2}$$

In the corporate finance literature, academics combine Modigliani and Miller's two equations to solve for the cost of equity—to demonstrate the relation between leverage and the cost of equity. In Appendix D, we algebraically rearrange equation 2 to solve for the levered cost of equity:

$$k_e = k_u + \frac{D}{E}(k_u - k_d) - \frac{V_{txa}}{E}(k_u - k_{txa})$$

As this equation indicates, the cost of equity depends on the unlevered cost of equity plus a premium for leverage, less a reduction for the tax deductibility of debt.

Determining the unlevered cost of equity with market data To use the APV, we need to discount projected free cash flow at the unlevered cost of equity, k_u. However, none of the variables (including k_u) on the left side of

equation 2 can be observed. Only the values on the right—that is, those related to debt and equity—can be estimated directly. Because there are so many unknowns and only one equation, we must impose additional restrictions to solve for k_u.

Method 1: Assume k_{txa} equals k_u If you believe the risk associated with tax shields (k_{txa}) equals the risk associated with operating assets (k_u), equation 2 can be simplified dramatically (see Appendix D):

$$k_u = \frac{E}{D+E} k_d + \frac{D}{D+E} k_e \tag{3}$$

We can now determine the unlevered cost of equity because it now relies solely on observable variables, that is, those related to debt and equity. In fact, k_u looks very similar to the weighted average cost of capital, without the interest tax shield.

Equation 3 can be rearranged to solve for the levered cost of equity:

$$k_e = k_u + \frac{D}{E}(k_u - k_d) \tag{4}$$

Note that when the company has no debt ($D = 0$), k_e equals k_u. This is why k_u is referred to as the unlevered cost of equity.

Method 2: Assume k_{txa} equals k_d If you believe the risk associated with tax shields (k_{txa}) is comparable to the risk of debt (k_d), equation 2 can be rearranged to solve for the unlevered cost of equity:

$$k_u = \frac{D - V_{txa}}{D - V_{txa} + E} k_d + \frac{E}{D - V_{txa} + E} k_e \tag{5}$$

In this equation, k_u relies on observable variables, such as the market value of debt, market value of equity, cost of debt, and cost of equity, as well as one unobservable variable: the present value of tax shields (V_{txa}). To use equation 4, discount expected future tax shields at the cost of debt (to remain consistent) and *then* solve for the unlevered cost of equity.

Many practitioners further refine the last equation by imposing an additional restriction: that the absolute dollar level of debt is constant. If the dollar level of debt is constant, the annual expected tax shield equals ($D \times k_d$) ×

T_m, where T_m equals the marginal tax rate. Applying a no-growth perpetuity formula allows us to value the tax shield:

$$V_{txa} = \frac{(D \times k_d) \times T_m}{k_d} = D \times T_m$$

Substituting $D \times T_m$ for the value of the tax shield in the last equation leads to:

$$k_u = \frac{(1-T_m)D}{(1-T_m)D+E} k_d + \frac{E}{(1-T_m)D+E} k_e \qquad (6)$$

Although equation 6 is quite common in practice, its use is limited because the assumptions are extremely restrictive.

Choosing the appropriate formula Which formula should you use to back-solve for the unlevered cost of equity, k_u? It depends on how you see the company managing its capital structure going forward and whether the debt is risk free. If you believe the company will manage its debt-to-value to a target level (the company's debt will grow with the business), then the value of the tax shields will track the value of the operating assets. Thus, the risk of tax shields will equal the risk of operating assets (k_{txa} equals k_u). The majority of companies have relatively stable capital structures (as a percentage of expected value), so we favor the first method.

If you believe the debt to equity ratio will not remain constant, then the value of interest tax shields will be more closely tied to the value of forecasted debt, rather than operating assets. In this case, the risk of tax shields is equivalent to the risk of debt (when a company is unprofitable, it cannot use interest tax shields, the risk of default rises, and the value of debt drops). In this case, equation 5 better approximates the unlevered cost of equity.[9] This situation occurs frequently in periods of high debt such as financial distress and leveraged buyouts.

Value Tax Shields and Other Capital Structure Effects

To complete an APV-based valuation, forecast and discount capital structure side effects such as tax shields, security issue costs, and distress costs. Since Home Depot has little chance of default, we estimated the company's future interest tax shields using the company's promised yield to maturity and marginal tax rate (see Exhibit 5.14). To calculate the expected interest

[9] Even if a company's tax shields are predetermined for a given period, eventually they will track value. For instance, successful leveraged buyouts pay down debt for a period of time, but once the debt level becomes reasonable, debt will more likely track value than remain constant.

Exhibit 5.14 Home Depot: Forecast of Interest Tax Shields

Year	Prior year net debt ($ million)	Expected interest rate (percent)	Interest payment ($ million)	Marginal tax rate (percent)	Interest tax shield ($ million)
2004	6,310	4.7	295	38.2	113
2005	6,737	4.7	315	38.2	120
2006	7,179	4.7	336	38.2	128
2007	7,637	4.7	357	38.2	136
2008	8,107	4.7	379	38.2	145
2009	8,589	4.7	402	38.2	153
2010	9,081	4.7	425	38.2	162
2011	9,579	4.7	448	38.2	171
2012	10,081	4.7	472	38.2	180
2013	10,583	4.7	495	38.2	189
Continuing value	11,082	4.7	518	38.2	198

payment in 2004, multiply the prior year's net debt of $6.3 billion by the expected yield of 4.7 percent (net debt equals reported debt plus capitalized operating leases minus excess cash). This led to an expected interest payment of $295 million. Next multiply the expected interest payment by the marginal tax rate of 38.2 percent, for an expected interest tax shield of $113 million in 2004.

Home Depot's conservative use of debt makes tax shield valuation straightforward. For companies with significant leverage, the company may not be able to fully use the tax shields (it may not have enough profits to shield). If there is a significant probability of default, you must model *expected* tax shields, rather than the tax shields based on promised interest payments. To do this, reduce each promised tax shield by the cumulative probability of default.

CAPITAL CASH FLOW MODEL

When a company actively manages its capital structure to a target debt-to-value level, both free cash flow (FCF) and the interest tax shield (ITS) are discounted at the unlevered cost of equity, k_u:

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+k_u)^t} + \sum_{t=1}^{\infty} \frac{ITS_t}{(1+k_u)^t}$$

In 2000, Richard Ruback of the Harvard Business School argued there is no need to separate free cash flow from tax shields when both flows are discounted by the same cost of capital.[10] He combined the two flows and named the resulting cash flow (FCF plus interest tax shields) capital cash flow (CCF):

$$V = \text{PV(Capital Cash Flows)} = \sum_{t=1}^{\infty} \frac{\text{FCF}_t + \text{ITS}_t}{(1+k_u)^t}$$

Given that Ruback's assumptions match those of the weighted average cost of capital, the capital cash flow and WACC-based valuations will lead to identical results. In fact, we now have detailed three distinct but identical valuation methods created solely around how they treat tax shields: WACC (tax shield valued in the cost of capital), APV (tax shield valued separately), and CCF (tax shield valued in the cash flow).

Although FCF and CCF lead to the same result when debt is proportional to value, we believe free cash flow models are superior to capital cash flow models. Why? By keeping NOPLAT and FCF independent of leverage, we can cleanly evaluate the company's operating performance over time and across competitors. A clean measure of historical operating performance leads to better forecasts.

CASH-FLOW-TO-EQUITY VALUATION MODEL

In each of the preceding valuation models, we determined the value of equity indirectly by subtracting nonequity claims from enterprise value. The equity cash flow model values equity directly by discounting cash flows to equity at the cost of equity, rather than at the weighted average cost of capital.[11]

Exhibit 5.15 details the cash flows to equity for Home Depot. Cash flow to equity can be computed by reorganizing free cash flow found in Exhibit 5.6 or using the traditional method in Exhibit 5.15. In the traditional method, cash flow to equity starts with net income. Next, noncash expenses are added back, and investments in working capital, fixed assets, and nonoperating assets are subtracted. Finally, any increases in nonequity financing such as debt are added, and decreases in nonequity financing are subtracted. Alternatively, we can compute cash flow to equity as dividends

[10] Richard S. Ruback, "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows," Social Science Research Network (March 2000).
[11] The equity method can be difficult to implement correctly because capital structure is embedded in the cash flow. This makes forecasting difficult. For companies whose operations are related to financing, such as financial institutions, the equity method is appropriate. We discuss valuing financial institutions in Chapter 25.

Exhibit 5.15 Home Depot: Equity Cash Flow Summary

$ million

	Historical			Forecast		
	2001	2002	2003	2004	2005	2006
Net income	3,044	3,664	4,304	4,796	5,318	5,882
Depreciation	756	895	1,075	1,193	1,321	1,459
Amortization	8	8	1	0	0	0
Increase (decrease) in deferred taxes	(6)	173	605	214	237	262
Gross cash flow	3,802	4,740	5,985	6,203	6,876	7,603
Investment in operating working capital	834	(194)	72	(294)	(318)	(344)
Investment in net long-term assets	(3,224)	(2,683)	(3,780)	(3,433)	(3,745)	(4,076)
Decrease (increase) in excess cash	(1,509)	383	(473)	(177)	(191)	(207)
Investment in other nonoperating assets	9	(24)	23	(9)	(10)	(11)
Increase (decrease) in short-term debt	207	(211)	509	(44)	(54)	(66)
Increase (decrease) in long-term debt	(295)	71	(465)	(73)	(91)	(112)
Cash flow to equity	(176)	2,082	1,871	2,173	2,466	2,788

	2001	2002	2003	2004	2005	2006
Dividends	396	492	595	663	735	813
Share repurchases (issued)	(572)	1,590	1,276	1,510	1,731	1,975
Cash flow to equity	(176)	2,082	1,871	2,173	2,466	2,788

plus share repurchases minus new equity issues. Both methods generate identical results.

To value Home Depot, we discount projected equity cash flows at the cost of equity (see Exhibit 5.16 on p. 128). Unlike enterprise-based models, no adjustments are made for nonoperating assets, debt, or capitalized operating leases. Rather, they are included as part of the equity cash flow.

Once again, note how the valuation, derived using equity cash flows, matches each of the prior valuations.[12] This occurs because we have modeled

[12] When performing a stand-alone equity cash flow valuation, you can calculate the continuing value by using a simple growing perpetuity:

$$V_e = \frac{\text{Net Income}\left(1 - \frac{g}{\text{ROE}}\right)}{k_e - g}$$

To tie the free cash flow and equity cash flow models, you must convert free cash flow continuing-value inputs into equity cash flow inputs. We did this using:

$$\text{Net Income}\left(1 - \frac{g}{\text{ROE}}\right) = \frac{\text{NOPLAT}\left(1 - \frac{g}{\text{ROIC}}\right)}{1 + \frac{D}{E}\left(1 - \frac{k_e - (1-T)k_d}{k_e - g}\right)}$$

Exhibit 5.16 Home Depot: Cash-Flow-to-Equity Valuation

Year	Cash flow to equity ($ million)	Discount factor (@ 9.9%)	Present value of CFE ($ million)
2004	2,173	0.910	1,978
2005	2,466	0.828	2,042
2006	2,788	0.754	2,101
2007	3,143	0.686	2,155
2008	3,530	0.624	2,203
2009	3,954	0.568	2,245
2010	4,416	0.517	2,282
2011	4,917	0.470	2,312
2012	5,459	0.428	2,336
2013	6,044	0.389	2,353
Continuing value	122,492	0.389	47,695
Present value of cash flow to equity			69,702
Midyear adjustment amount			3,456
Equity value			**73,158**

Home Depot's debt-to-value ratio at a constant level. If debt-to-value instead changes over time, the equity model becomes difficult to implement and can lead to conceptual errors. For example, if leverage is expected to rise, the cost of equity must be adjusted to reflect the additional risk imposed on equity holders. Although formulas exist to adjust the cost of equity, many of the best-known formulas are built under restrictions that may be inconsistent with the way you are implicitly forecasting the company's capital structure via the cash flows. This will cause a mismatch between cash flows and the cost of equity, resulting in an incorrect valuation.

Unwittingly changing the company's capital structure when using the cash-flow-to-equity model occurs too easily—and that is what makes the model so risky. Suppose you plan to value a company whose debt-to-value ratio is 15 percent. You believe the company will pay extra dividends, so you increase debt to raise the dividend payout ratio. Presto! Increased dividends lead to higher equity cash flows and a higher valuation. Even though operating performance has not changed, the equity value has mistakenly increased. What happened? Using new debt to pay dividends causes a rise in net debt to value. Unless you adjust the cost of equity, the valuation will rise incorrectly.

Another shortcoming of the direct equity approach occurs when valuing a company by business unit. The direct equity approach requires allocating debt and interest expense to each unit. This creates extra work yet provides few additional insights.

OTHER APPROACHES TO DISCOUNTED CASH FLOW

You may also come across two variants of enterprise DCF:

1. Using real instead of nominal cash flows and discount rates
2. Discounting pretax cash flows instead of after-tax cash flows

These approaches are well suited only to limited circumstances.

Using Real Cash Flows and Discount Rates

Companies can be valued by projecting cash flow in real terms (e.g., in constant 2004 dollars) and discounting this cash flow at a real discount rate (e.g., the nominal rate less expected inflation). But most managers think in terms of nominal rather than real measures, so nominal measures are often easier to communicate. In addition, interest rates are generally quoted nominally rather than in real terms (excluding expected inflation). Also, since historical financial statements are stated in nominal terms, projecting future statements in real terms is difficult and confusing.

A second difficulty occurs when calculating and interpreting ROIC. The historical statements are nominal, so historical returns on invested capital are nominal. But if the projections for the company use real rather than nominal forecasts, returns on new capital are also real. Projected returns on total capital (new and old) are a combination of nominal and real, so they are impossible to interpret. The only way around this is to restate historical performance on a real basis. This is a complex and time-consuming task. The extra insights gained rarely equal the effort (except in extremely high-inflation environments described in Chapter 22).

Discounting Pretax Cash Flow

For purposes of valuing internal investment opportunities, individual project cash flows are sometimes calculated without taxes. The pretax cash flow is then discounted by a pretax "hurdle rate" (the market-based cost of capital multiplied by 1 plus the marginal tax rate) to determine a pretax value.

This method, however, leads to three fundamental inconsistencies. First, the government calculates taxes on profits after depreciation, not on cash flow after capital expenditures. By discounting pretax cash flow at the pretax cost of capital, you implicitly assume capital investments are tax deductible when made, not as they are depreciated. Furthermore, short-term investments, such as accounts receivable and inventory, are never tax deductible. Selling a product at a profit is what leads to incremental taxes, not holding inventory. By discounting pretax cash flow at the pretax cost of capital, you incorrectly assume investments in operating working capital are tax deductible. Finally, it can be shown that even when net investment equals depreciation, the final result will be downward biased—and the larger the cost of capital, the larger the bias. This bias occurs because the method is only an approximation, not a formal mathematical relation. Because of these inconsistencies, we recommend against discounting pretax cash flows at a pretax hurdle rate.

ALTERNATIVES TO DISCOUNTED CASH FLOW

To this point, we have focused solely on discounted cash flow models. Two additional valuation techniques exist: multiples (comparables) and real options.

Multiples

Assume that you have been asked to value a company that is about to go public. Although you project and discount free cash flow to derive an enterprise value, you worry that your forecasts lack precision. One way to place your DCF model in the proper context is to create a set of comparables. One of the most commonly used comparables is the enterprise-value-to-earnings before interest, taxes, and amortization (EV/EBITA) multiple. To apply the EV/EBITA multiple, look for a set of comparable companies, and multiply a representative EV/EBITA multiple by the company's EBITA. For example, assume the company's EBITA equals $100 million and the typical EV/EBITA multiple in the industry is 15×. Multiplying 15 by $100 million leads to an estimated value of $1.5 billion. Is the enterprise DCF valuation near $1.5 billion? If not, what enables the company to earn better (or worse) returns or grow faster (or slower) than other companies in the industry?

Although the concept of multiples is simple, the methodology is misunderstood and often misapplied. Companies within an industry will have different multiples for valid economic reasons. Computing a representative multiple ignores this fact. In addition, common multiples, such as the price-to-earnings ratio, suffer from the same capital structure problems as equity cash flows. In Chapter 12, we demonstrate how to build and interpret forward-looking comparables, independent of capital structure and other nonoperating items.

Real Options

In 1997 Robert Merton and Myron Scholes won the Nobel Prize in Economics for developing an ingenious method to value derivatives that avoids the need to estimate either cash flows or the cost of capital. (Fisher Black would

have been named as a third recipient, but the Nobel Prize is not awarded posthumously.) Their model relies on what today's economists call a "replicating portfolio." They argued that if there exists a portfolio of traded securities whose future cash flows perfectly mimic the security you are attempting to value, the portfolio and security must have the same price. As long as we can find a suitable replicating portfolio, we need not discount future cash flows.

Given the model's power, there have been many recent attempts to translate the concepts of replicating portfolios to corporate valuation. This valuation technique is commonly known as real options. Unlike those for financial options, however, replicating portfolios for companies and their projects may be difficult to create. Therefore, although options-pricing models may teach powerful lessons, today's applications are limited. We cover valuation using options-based models in Chapter 20.

SUMMARY

This chapter described the most common DCF valuation models, with particular focus on the enterprise DCF model and the economic profit model. We explained the rationale for each model and reasons why each model has an important place in corporate valuation. The remaining chapters in Part Two describe a step-by-step approach to valuing a company:

- Chapter 6: Thinking about Return on Invested Capital and Growth
- Chapter 7: Analyzing Historical Performance
- Chapter 8: Forecasting Performance
- Chapter 9: Estimating Continuing Value
- Chapter 10: Estimating the Cost of Capital
- Chapter 11: Calculating and Interpreting Results
- Chapter 12: Using Multiples for Valuation

These chapters explain the technical details of valuation, including how to calculate free cash flow from the accounting statements and how to create and interpret the valuation through careful financial analysis.

The American Economic Review

VOLUME XLVIII JUNE 1958 NUMBER THREE

THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

By FRANCO MODIGLIANI AND MERTON H. MILLER*

What is the "cost of capital" to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be obtained by many different media, ranging from pure debt instruments, representing money-fixed claims, to pure equity issues, giving holders only the right to a pro-rata share in the uncertain venture? This question has vexed at least three classes of economists: (1) the corporation finance specialist concerned with the techniques of financing firms so as to ensure their survival and growth; (2) the managerial economist concerned with capital budgeting; and (3) the economic theorist concerned with explaining investment behavior at both the micro and macro levels.[1]

In much of his formal analysis, the economic theorist at least has tended to side-step the essence of this cost-of-capital problem by proceeding as though physical assets—like bonds—could be regarded as yielding known, sure streams. Given this assumption, the theorist has concluded that the cost of capital to the owners of a firm is simply the rate of interest on bonds; and has derived the familiar proposition that the firm, acting rationally, will tend to push investment to the point

* The authors are, respectively, professor and associate professor of economics in the Graduate School of Industrial Administration, Carnegie Institute of Technology. This article is a revised version of a paper delivered at the annual meeting of the Econometric Society, December 1956. The authors express thanks for the comments and suggestions made at that time by the discussants of the paper, Evsey Domar, Robert Eisner and John Lintner, and subsequently by James Duesenberry. They are also greatly indebted to many of their present and former colleagues and students at Carnegie Tech who served so often and with such remarkable patience as a critical forum for the ideas here presented.

[1] The literature bearing on the cost-of-capital problem is far too extensive for listing here. Numerous references to it will be found throughout the paper though we make no claim to completeness. One phase of the problem which we do not consider explicitly, but which has a considerable literature of its own is the relation between the cost of capital and public utility rates. For a recent summary of the "cost-of-capital theory" of rate regulation and a brief discussion of some of its implications, the reader may refer to H. M. Somers [20].

where the marginal yield on physical assets is equal to the market rate of interest.[2] This proposition can be shown to follow from either of two criteria of rational decision-making which are equivalent under certainty, namely (1) the maximization of profits and (2) the maximization of market value.

According to the first criterion, a physical asset is worth acquiring if it will increase the net profit of the owners of the firm. But net profit will increase only if the expected rate of return, or yield, of the asset exceeds the rate of interest. According to the second criterion, an asset is worth acquiring if it increases the value of the owners' equity, *i.e.*, if it adds more to the market value of the firm than the costs of acquisition. But what the asset adds is given by capitalizing the stream it generates at the market rate of interest, and this capitalized value will exceed its cost if and only if the yield of the asset exceeds the rate of interest. Note that, under either formulation, the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.

It must be acknowledged that some attempt is usually made in this type of analysis to allow for the existence of uncertainty. This attempt typically takes the form of superimposing on the results of the certainty analysis the notion of a "risk discount" to be subtracted from the expected yield (or a "risk premium" to be added to the market rate of interest). Investment decisions are then supposed to be based on a comparison of this "risk adjusted" or "certainty equivalent" yield with the market rate of interest.[3] No satisfactory explanation has yet been provided, however, as to what determines the size of the risk discount and how it varies in response to changes in other variables.

Considered as a convenient approximation, the model of the firm constructed via this certainty—or certainty-equivalent—approach has admittedly been useful in dealing with some of the grosser aspects of the processes of capital accumulation and economic fluctuations. Such a model underlies, for example, the familiar Keynesian aggregate investment function in which aggregate investment is written as a function of the rate of interest—the same riskless rate of interest which appears later in the system in the liquidity-preference equation. Yet few would maintain that this approximation is adequate. At the macroeconomic level there are ample grounds for doubting that the rate of interest has

[2] Or, more accurately, to the marginal cost of borrowed funds since it is customary, at least in advanced analysis, to draw the supply curve of borrowed funds to the firm as a rising one. For an advanced treatment of the certainty case, see F. and V. Lutz [13].

[3] The classic examples of the certainty-equivalent approach are found in J. R. Hicks [8] and O. Lange [11].

as large and as direct an influence on the rate of investment as this analysis would lead us to believe. At the microeconomic level the certainty model has little descriptive value and provides no real guidance to the finance specialist or managerial economist whose main problems cannot be treated in a framework which deals so cavalierly with uncertainty and ignores all forms of financing other than debt issues.[4]

Only recently have economists begun to face up seriously to the problem of the cost of capital *cum* risk. In the process they have found their interests and endeavors merging with those of the finance specialist and the managerial economist who have lived with the problem longer and more intimately. In this joint search to establish the principles which govern rational investment and financial policy in a world of uncertainty two main lines of attack can be discerned. These lines represent, in effect, attempts to extrapolate to the world of uncertainty each of the two criteria—profit maximization and market value maximization—which were seen to have equivalent implications in the special case of certainty. With the recognition of uncertainty this equivalence vanishes. In fact, the profit maximization criterion is no longer even well defined. Under uncertainty there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has become a random variable and as such its maximization no longer has an operational meaning. Nor can this difficulty generally be disposed of by using the mathematical expectation of profits as the variable to be maximized. For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes.

Under these conditions the profit outcomes of alternative investment and financing decisions can be compared and ranked only in terms of a *subjective* "utility function" of the owners which weighs the expected yield against other characteristics of the distribution. Accordingly, the extrapolation of the profit maximization criterion of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form.[5]

The utility approach undoubtedly represents an advance over the certainty or certainty-equivalent approach. It does at least permit us

[4] Those who have taken a "case-method" course in finance in recent years will recall in this connection the famous Liquigas case of Hunt and Williams, [9, pp. 193–96] a case which is often used to introduce the student to the cost-of-capital problem and to poke a bit of fun at the economist's certainty-model.

[5] For an attempt at a rigorous explicit development of this line of attack, see F. Modigliani and M. Zeman [14].

to explore (within limits) some of the implications of different financing arrangements, and it does give some meaning to the "cost" of different types of funds. However, because the cost of capital has become an essentially subjective concept, the utility approach has serious drawbacks for normative as well as analytical purposes. How, for example, is management to ascertain the risk preferences of its stockholders and to compromise among their tastes? And how can the economist build a meaningful investment function in the face of the fact that any given investment opportunity might or might not be worth exploiting depending on precisely who happen to be the owners of the firm at the moment?

Fortunately, these questions do not have to be answered; for the alternative approach, based on market value maximization, can provide the basis for an operational definition of the cost of capital and a workable theory of investment. Under this approach any investment project and its concomitant financing plan must pass only the following test: Will the project, as financed, raise the market value of the firm's shares? If so, it is worth undertaking; if not, its return is less than the marginal cost of capital to the firm. Note that such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well. If any current stockholder disagrees with management and the market over the valuation of the project, he is free to sell out and reinvest elsewhere, but will still benefit from the capital appreciation resulting from management's decision.

The potential advantages of the market-value approach have long been appreciated; yet analytical results have been meager. What appears to be keeping this line of development from achieving its promise is largely the lack of an adequate theory of the effect of financial structure on market valuations, and of how these effects can be inferred from objective market data. It is with the development of such a theory and of its implications for the cost-of-capital problem that we shall be concerned in this paper.

Our procedure will be to develop in Section I the basic theory itself and to give some brief account of its empirical relevance. In Section II, we show how the theory can be used to answer the cost-of-capital question and how it permits us to develop a theory of investment of the firm under conditions of uncertainty. Throughout these sections the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given. We have chosen to focus at this level rather than on the economy as a whole because it is at the level of the firm and the industry that the interests of the various specialists concerned with the cost-of-capital problem come most closely together. Although the emphasis has thus been placed on partial-equilibrium analysis, the results obtained also provide the essential building blocks for a general equilibrium model which shows how those prices which are here taken as given, are themselves determined. For reasons of space, however, and because the material is of interest in its own right, the presentation of the general equilibrium model which rounds out the analysis must be deferred to a subsequent paper.

I. *The Valuation of Securities, Leverage, and the Cost of Capital*

A. *The Capitalization Rate for Uncertain Streams*

As a starting point, consider an economy in which all physical assets are owned by corporations. For the moment, assume that these corporations can finance their assets by issuing common stock only; the introduction of bond issues, or their equivalent, as a source of corporate funds is postponed until the next part of this section.

The physical assets held by each firm will yield to the owners of the firm—its stockholders—a stream of "profits" over time; but the elements of this series need not be constant and in any event are uncertain. This stream of income, and hence the stream accruing to any share of common stock, will be regarded as extending indefinitely into the future. We assume, however, that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution. We shall refer to the average value over time of the stream accruing to a given share as the return of that share; and to the mathematical expectation of this average as the expected return of the share.[6] Although individual investors may have different views as to the shape of the probability distri-

[6] These propositions can be restated analytically as follows: The assets of the ith firm generate a stream:

$$X_i(1),\ X_i(2) \cdots X_i(T)$$

whose elements are random variables subject to the joint probability distribution:

$$\chi_i[X_i(1),\ X_i(2) \cdots X_i(t)].$$

The return to the ith firm is defined as:

$$X_i = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} X_i(t).$$

X_i is itself a random variable with a probability distribution $\Phi_i(X_i)$ whose form is determined uniquely by χ_i. The expected return $\bar{X}_i$ is defined as $\bar{X}_i = E(X_i) = \int_{X_i} X_i \Phi_i(X_i)\, dX_i$. If N_i is the number of shares outstanding, the return of the ith share is $x_i = (1/N)X_i$ with probability distribution $\phi_i(x_i)dx_i = \Phi_i(Nx_i)d(Nx_i)$ and expected value $\bar{x}_i = (1/N)\bar{X}_i$.

bution of the return of any share, we shall assume for simplicity that they are at least in agreement as to the expected return.[7]

This way of characterizing uncertain streams merits brief comment. Notice first that the stream is a stream of profits, not dividends. As will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail. Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two totally different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. It can be shown, furthermore, that whether the elements of a stream are sure or uncertain, the effect of variability per se on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too).[8]

The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the *ratio* of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return.

The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another. We have, thus, an analogue to the familiar concept of the industry in which it is the commodity produced by the firms that is taken as homogeneous. To complete this analogy with Marshallian price theory, we shall assume in the

[7] To deal adequately with refinements such as differences among investors in estimates of expected returns would require extensive discussion of the theory of portfolio selection. Brief references to these and related topics will be made in the succeeding article on the general equilibrium model.

[8] The reader may convince himself of this by asking how much he would be willing to rebate to his employer for the privilege of receiving his annual salary in equal monthly installments rather than in irregular amounts over the year. See also J. M. Keynes [10, esp. pp. 53–54].

analysis to follow that the shares concerned are traded in perfect markets under conditions of atomistic competition.[9]

From our definition of homogeneous classes of stock it follows that in equilibrium in a perfect capital market the price per dollar's worth of expected return must be the same for all shares of any given class. Or, equivalently, in any given class the price of every share must be proportional to its expected return. Let us denote this factor of proportionality for any class, say the kth class, by $1/\rho_k$. Then if p_j denotes the price and $\bar{x}_j$ is the expected return per share of the jth firm in class k, we must have:

$$p_j = \frac{1}{\rho_k}\bar{x}_j; \tag{1}$$

or, equivalently,

$$\frac{\bar{x}_j}{p_j} = \rho_k \text{ a constant for all firms } j \text{ in class } k. \tag{2}$$

The constants ρ_k (one for each of the k classes) can be given several economic interpretations: (a) From (2) we see that each ρ_k is the expected rate of return of any share in class k. (b) From (1) $1/\rho_k$ is the price which an investor has to pay for a dollar's worth of expected return in the class k. (c) Again from (1), by analogy with the terminology for perpetual bonds, ρ_k can be regarded as the market rate of capitalization for the expected value of the uncertain streams of the kind generated by the kth class of firms.[10]

B. *Debt Financing and Its Effects on Security Prices*

Having developed an apparatus for dealing with uncertain streams we can now approach the heart of the cost-of-capital problem by dropping the assumption that firms cannot issue bonds. The introduction of debt-financing changes the market for shares in a very fundamental way. Because firms may have different proportions of debt in their capi-

[9] Just what our classes of stocks contain and how the different classes can be identified by outside observers are empirical questions to which we shall return later. For the present, it is sufficient to observe: (1) Our concept of a class, while not identical to that of the industry is at least closely related to it. Certainly the basic characteristics of the probability distributions of the returns on assets will depend to a significant extent on the product sold and the technology used. (2) What are the appropriate class boundaries will depend on the particular problem being studied. An economist concerned with general tendencies in the market, for example, might well be prepared to work with far wider classes than would be appropriate for an investor planning his portfolio, or a firm planning its financial strategy.

[10] We cannot, on the basis of the assumptions so far, make any statements about the relationship or spread between the various ρ's or capitalization rates. Before we could do so we would have to make further specific assumptions about the way investors believe the probability distributions vary from class to class, as well as assumptions about investors' preferences as between the characteristics of different distributions.

tal structure, shares of different companies, even in the same class, can give rise to different probability distributions of returns. In the language of finance, the shares will be subject to different degrees of financial risk or "leverage" and hence they will no longer be perfect substitutes for one another.

To exhibit the mechanism determining the relative prices of shares under these conditions, we make the following two assumptions about the nature of bonds and the bond market, though they are actually stronger than is necessary and will be relaxed later: (1) All bonds (including any debts issued by households for the purpose of carrying shares) are assumed to yield a constant income per unit of time, and this income is regarded as certain by all traders regardless of the issuer. (2) Bonds, like stocks, are traded in a perfect market, where the term perfect is to be taken in its usual sense as implying that any two commodities which are perfect substitutes for each other must sell, in equilibrium, at the same price. It follows from assumption (1) that all bonds are in fact perfect substitutes up to a scale factor. It follows from assumption (2) that they must all sell at the same price per dollar's worth of return, or what amounts to the same thing must yield the same rate of return. This rate of return will be denoted by r and referred to as the rate of interest or, equivalently, as the capitalization rate for sure streams. We now can derive the following two basic propositions with respect to the valuation of securities in companies with different capital structures:

Proposition I. Consider any company j and let $\overline{X}_j$ stand as before for the expected return on the assets owned by the company (that is, its expected profit before deduction of interest). Denote by D_j the market value of the debts of the company; by S_j the market value of its common shares; and by $V_j \equiv S_j + D_j$ the market value of all its securities or, as we shall say, the market value of the firm. Then, our Proposition I asserts that we must have in equilibrium:

$$(3) \qquad V_j \equiv (S_j + D_j) = \overline{X}_j/\rho_k, \text{ for any firm } j \text{ in class } k.$$

That is, the *market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ_k appropriate to its class.*

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\overline{X}_j/V_j$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$(4) \qquad \frac{\overline{X}_j}{(S_j + D_j)} \equiv \frac{\overline{X}_j}{V_j} = \rho_k, \text{ for any firm } j, \text{ in class } k.$$

That is, *the average cost of capital to any firm is completely independent of* *its capital structure and is equal to the capitalization rate of a pure equity stream of its class.*

To establish Proposition I we will show that as long as the relations (3) or (4) do not hold between any pair of firms in a class, arbitrage will take place and restore the stated equalities. We use the term arbitrage advisedly. For if Proposition I did not hold, an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price. The exchange would therefore be advantageous to the investor quite independently of his attitudes toward risk.[11] As investors exploit these arbitrage opportunities, the value of the overpriced shares will fall and that of the underpriced shares will rise, thereby tending to eliminate the discrepancy between the market values of the firms.

By way of proof, consider two firms in the same class and assume for simplicity only, that the expected return, $\bar{X}$, is the same for both firms. Let company 1 be financed entirely with common stock while company 2 has some debt in its capital structure. Suppose first the value of the levered firm, V_2, to be larger than that of the unlevered one, V_1. Consider an investor holding s_2 dollars' worth of the shares of company 2, representing a fraction α of the total outstanding stock, S_2. The return from this portfolio, denoted by Y_2, will be a fraction α of the income available for the stockholders of company 2, which is equal to the total return X_2 less the interest charge, rD_2. Since under our assumption of homogeneity, the anticipated total return of company 2, X_2, is, under all circumstances, the same as the anticipated total return to company 1, X_1, we can hereafter replace X_2 and X_1 by a common symbol X. Hence, the return from the initial portfolio can be written as:

$$Y_2 = \alpha(X - rD_2). \tag{5}$$

Now suppose the investor sold his αS_2 worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount αS_2 realized from the sale of his initial holding and borrowing an additional amount αD_2 on his own credit, pledging his new holdings in company 1 as a collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares and earnings of company 1. Making proper allowance for the interest payments on his personal debt αD_2, the return from the new portfolio, Y_1, is given by:

[11] In the language of the theory of choice, the exchanges are movements from inefficient points in the interior to efficient points on the boundary of the investor's opportunity set; and not movements between efficient points along the boundary. Hence for this part of the analysis nothing is involved in the way of specific assumptions about investor attitudes or behavior other than that investors behave consistently and prefer more income to less income, *ceteris paribus*.

$$(6) \qquad Y_1 = \frac{\alpha(S_2 + D_2)}{S_1} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2.$$

Comparing (5) with (6) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of company 2's shares to sell their holdings, thereby depressing S_2 and hence V_2; and to acquire shares of company 1, thereby raising S_1 and thus V_1. We conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account.

Consider now the other possibility, namely that the market value of the levered company V_2 is less than V_1. Suppose an investor holds initially an amount s_1 of shares of company 1, representing a fraction α of the total outstanding stock, S_1. His return from this holding is:

$$Y_1 = \frac{s_1}{S_1} X = \alpha X.$$

Suppose he were to exchange this initial holding for another portfolio, also worth s_1, but consisting of s_2 dollars of stock of company 2 and of d dollars of bonds, where s_2 and d are given by:

$$(7) \qquad s_2 = \frac{S_2}{V_2} s_1, \qquad d = \frac{D_2}{V_2} s_1.$$

In other words the new portfolio is to consist of stock of company 2 and of bonds in the proportions S_2/V_2 and D_2/V_2, respectively. The return from the stock in the new portfolio will be a fraction s_2/S_2 of the total return to stockholders of company 2, which is $(X - rD_2)$, and the return from the bonds will be rd. Making use of (7), the total return from the portfolio, Y_2, can be expressed as follows:

$$Y_2 = \frac{s_2}{S_2}(X - rD_2) + rd = \frac{s_1}{V_2}(X - rD_2) + r\frac{D_2}{V_2} s_1 = \frac{s_1}{V_2} X = \alpha \frac{S_1}{V_2} X$$

(since $s_1 = \alpha S_1$). Comparing Y_2 with Y_1 we see that, if $V_2 < S_1 \equiv V_1$, then Y_2 will exceed Y_1. Hence it pays the holders of company 1's shares to sell these holdings and replace them with a mixed portfolio containing an appropriate fraction of the shares of company 2.

The acquisition of a mixed portfolio of stock of a levered company j and of bonds in the proportion S_j/V_j and D_j/V_j respectively, may be regarded as an operation which "undoes" the leverage, giving access to an appropriate fraction of the unlevered return X_j. It is this possibility of undoing leverage which prevents the value of levered firms from being consistently less than those of unlevered firms, or more generally prevents the average cost of capital $\overline{X}_j/V_j$ from being systematically higher for levered than for nonlevered companies in the same class.

Since we have already shown that arbitrage will also prevent V_2 from being larger than V_1, we can conclude that in equilibrium we must have $V_2 = V_1$, as stated in Proposition I.

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, i, on the stock of any company j belonging to the kth class is a linear function of leverage as follows:

$$i_j = \rho_k + (\rho_k - r)D_j/S_j. \tag{8}$$

That is, *the expected yield of a share of stock is equal to the appropriate capitalization rate ρ_k for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ_k and r.* Or equivalently, the market price of any share of stock is given by capitalizing its expected return at the continuously variable rate i_j of (8).[12]

A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results were not applicable to the actual capital markets.[13] Proposition II, however, so far as we have been able to discover is new.[14] To establish it we first note that, by definition, the expected rate of return, i, is given by:

$$i_j \equiv \frac{\overline{X}_j - rD_j}{S_j}. \tag{9}$$

From Proposition I, equation (3), we know that:

$$\overline{X}_j = \rho_k(S_j + D_j).$$

Substituting in (9) and simplifying, we obtain equation (8).

[12] To illustrate, suppose $\overline{X} = 1000$, $D = 4000$, $r = 5$ per cent and $\rho_k = 10$ per cent. These values imply that $V = 10{,}000$ and $S = 6000$ by virtue of Proposition I. The expected yield or rate of return per share is then:

$$i = \frac{1000 - 200}{6000} = .1 + (.1 - .05)\frac{4000}{6000} = 13\tfrac{1}{3} \text{ per cent.}$$

[13] See, for example, J. B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their "riskiness." This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial. Regarding Proposition I as resting on investors' attitudes toward risk leads inevitably to a misunderstanding of many factors influencing relative yields such as, for example, limitations on the portfolio composition of financial institutions. See below, esp. Section I.D.

[14] Morton does make reference to a linear yield function but only ". . . for the sake of simplicity and because the particular function used makes no essential difference in my conclusions" [15, p. 443, note 2].

C. *Some Qualifications and Extensions of the Basic Propositions*

The methods and results developed so far can be extended in a number of useful directions, of which we shall consider here only three: (1) allowing for a corporate profits tax under which interest payments are deductible; (2) recognizing the existence of a multiplicity of bonds and interest rates; and (3) acknowledging the presence of market imperfections which might interfere with the process of arbitrage. The first two will be examined briefly in this section with some further attention given to the tax problem in Section II. Market imperfections will be discussed in Part D of this section in the course of a comparison of our results with those of received doctrines in the field of finance.

Effects of the Present Method of Taxing Corporations. The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that the market values of firms in each class must be proportional in equilibrium to their expected return net of taxes (that is, to the sum of the interest paid and expected net stockholder income). This means we must replace each $\bar{X}_j$ in the original versions of Propositions I and II with a new variable $\bar{X}_j^\tau$ representing the total income net of taxes generated by the firm:

$$(10) \qquad \bar{X}_j^\tau \equiv (\bar{X}_j - rD_j)(1 - \tau) + rD_j \equiv \bar{\pi}_j^\tau + rD_j,$$

where $\bar{\pi}_j^\tau$ represents the expected net income accruing to the common stockholders and τ stands for the average rate of corporate income tax.[15]

After making these substitutions, the propositions, when adjusted for taxes, continue to have the same form as their originals. That is, Proposition I becomes:

$$(11) \qquad \frac{\bar{X}_j^\tau}{V_j} = \rho_k^\tau, \text{ for any firm in class } k,$$

and Proposition II becomes

$$(12) \qquad i_j \equiv \frac{\bar{\pi}_j^\tau}{S_j} = \rho_j^\tau + (\rho_k^\tau - r)D_j/S_j$$

where ρ_k^τ is the capitalization rate for income net of taxes in class k.

Although the form of the propositions is unaffected, certain interpretations must be changed. In particular, the after-tax capitalization rate

[15] For simplicity, we shall ignore throughout the tiny element of progression in our present corporate tax and treat τ as a constant independent of $(X_j - rD_j)$.

ρ_k^τ can no longer be identified with the "average cost of capital" which is $\rho_k = \bar{X}_j/V_j$. The difference between ρ_k^τ and the "true" average cost of capital, as we shall see, is a matter of some relevance in connection with investment planning within the firm (Section II). For the description of market behavior, however, which is our immediate concern here, the distinction is not essential. To simplify presentation, therefore, and to preserve continuity with the terminology in the standard literature we shall continue in this section to refer to ρ_k^τ as the average cost of capital, though strictly speaking this identification is correct only in the absence of taxes.

Effects of a Plurality of Bonds and Interest Rates. In existing capital markets we find not one, but a whole family of interest rates varying with maturity, with the technical provisions of the loan and, what is most relevant for present purposes, with the financial condition of the borrower.[16] Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio of the borrowing firm (or individual). If so, and if we can assume as a first approximation that this yield curve, $r = r\ (D/S)$, whatever its precise form, is the same for all borrowers, then we can readily extend our propositions to the case of a rising supply curve for borrowed funds.[17]

Proposition I is actually unaffected in form and interpretation by the fact that the rate of interest may rise with leverage; while the average cost of *borrowed* funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage (apart from the tax effect). This conclusion follows directly from the ability of those who engage in arbitrage to undo the leverage in any financial structure by acquiring an appropriately mixed portfolio of bonds and stocks. Because of this ability, the ratio of earnings (*before* interest charges) to market value—*i.e.*, the average cost of capital from all

[16] We shall not consider here the extension of the analysis to encompass the time structure of interest rates. Although some of the problems posed by the time structure can be handled within our comparative statics framework, an adequate discussion would require a separate paper.

[17] We can also develop a theory of bond valuation along lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be a nonlinear one in contrast to the linear function of leverage developed for common shares. However, we would also expect that the rate of increase in the yield on new issues would not be substantial in practice. This relatively slow rise would reflect the fact that interest rate increases by themselves can never be completely satisfactory to creditors as compensation for their increased risk. Such increases may simply serve to raise r so high relative to ρ that they become self-defeating by giving rise to a situation in which even normal fluctuations in earnings may force the company into bankruptcy. The difficulty of borrowing more, therefore, tends to show up in the usual case not so much in higher rates as in the form of increasingly stringent restrictions imposed on the company's management and finances by the creditors; and ultimately in a complete inability to obtain new borrowed funds, at least from the institutional investors who normally set the standards in the market for bonds.

sources—must be the same for all firms in a given class.[18] In other words, the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock. This seemingly paradoxical result will be examined more closely below in connection with Proposition II.

A significant modification of Proposition I would be required only if the yield curve $r=r(D/S)$ were different for different borrowers, as might happen if creditors had marked preferences for the securities of a particular class of debtors. If, for example, corporations as a class were able to borrow at lower rates than individuals having equivalent personal leverage, then the average cost of capital to corporations might fall slightly, as leverage increased over some range, in reflection of this differential. In evaluating this possibility, however, remember that the relevant interest rate for our arbitrage operators is the rate on brokers' loans and, historically, that rate has not been noticeably higher than representative corporate rates.[19] The operations of holding companies and investment trusts which can borrow on terms comparable to operating companies represent still another force which could be expected to wipe out any marked or prolonged advantages from holding levered stocks.[20]

Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If r increases with leverage, the yield i will still tend to

[18] One normally minor qualification might be noted. Once we relax the assumption that all bonds have certain yields, our arbitrage operator faces the danger of something comparable to "gambler's ruin." That is, there is always the possibility that an otherwise sound concern—one whose long-run expected income is greater than its interest liability—might be forced into liquidation as a result of a run of temporary losses. Since reorganization generally involves costs, and because the operation of the firm may be hampered during the period of reorganization with lasting unfavorable effects on earnings prospects, we might perhaps expect heavily levered companies to sell at a slight discount relative to less heavily indebted companies of the same class.

[19] Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form, not of having the arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock. Margin requirements are also somewhat less of an obstacle to maintaining any desired degree of leverage in a portfolio than might be thought at first glance. Leverage could be largely restored in the face of higher margin requirements by switching to stocks having more leverage at the corporate level.

[20] An extreme form of inequality between borrowing and lending rates occurs, of course, in the case of preferred stocks, which can not be directly issued by individuals on personal account. Here again, however, we would expect that the operations of investment corporations plus the ability of arbitrage operators to sell off their holdings of preferred stocks would act to prevent the emergence of any substantial premiums (for this reason) on capital structures containing preferred stocks. Nor are preferred stocks so far removed from bonds as to make it impossible for arbitrage operators to approximate closely the risk and leverage of a corporate preferred stock by incurring a somewhat smaller debt on personal account.

rise as D/S increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to fall.[21] The relation between i and D/S could conceivably take the form indicated by the curve MD

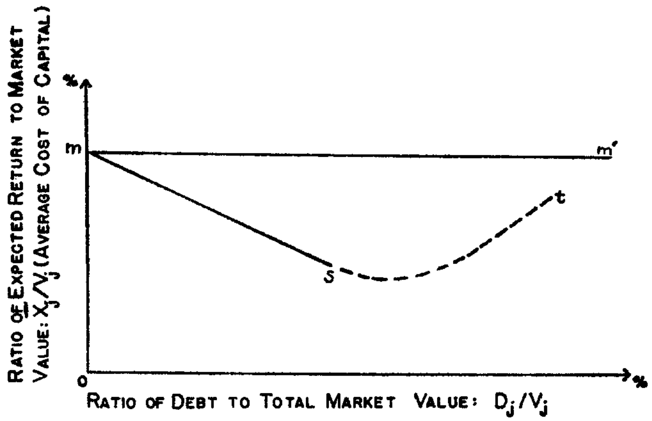


FIGURE 1

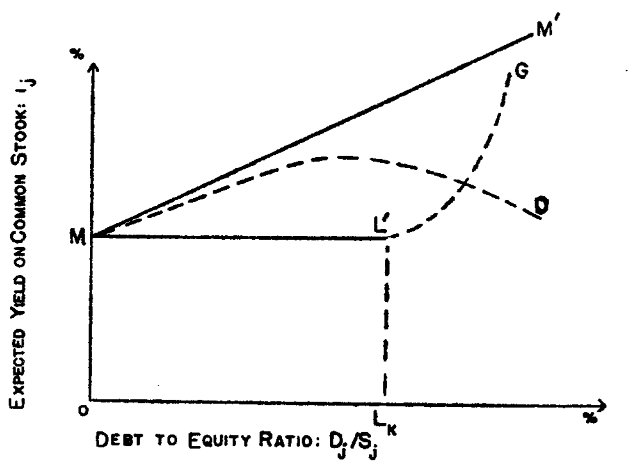


FIGURE 2

in Figure 2, although in practice the curvature would be much less pronounced. By contrast, with a constant rate of interest, the relation would be linear throughout as shown by line MM', Figure 2.

The downward sloping part of the curve MD perhaps requires some

[21] Since new lenders are unlikely to permit this much leverage (*cf.* note 17), this range of the curve is likely to be occupied by companies whose earnings prospects have fallen substantially since the time when their debts were issued.

comment since it may be hard to imagine why investors, other than those who like lotteries, would purchase stocks in this range. Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve MD, this demand would be reinforced by the action of arbitrage operators. The latter would find it profitable to own a pro-rata share of the firm as a whole by holding its stock *and* bonds, the lower yield of the shares being thus offset by the higher return on bonds.

D. *The Relation of Propositions I and II to Current Doctrines*

The propositions we have developed with respect to the valuation of firms and shares appear to be substantially at variance with current doctrines in the field of finance. The main differences between our view and the current view are summarized graphically in Figures 1 and 2. Our Proposition I [equation (4)] asserts that the average cost of capital, $\overline{X}_j^\tau/V_j$, is a constant for all firms j in class k, independently of their financial structure. This implies that, if we were to take a sample of firms in a given class, and if for each firm we were to plot the ratio of expected return to market value against some measure of leverage or financial structure, the points would tend to fall on a horizontal straight line with intercept ρ_k^τ, like the solid line mm' in Figure 1.[22] From Proposition I we derived Proposition II [equation (8)] which, taking the simplest version with r constant, asserts that, for all firms in a class, the relation between the yield on common stock and financial structure, measured by D_j/S_j, will approximate a straight line with slope $(\rho_k^\tau - r)$ and intercept ρ_k^τ. This relationship is shown as the solid line MM' in Figure 2, to which reference has been made earlier.[23]

By contrast, the conventional view among finance specialists appears to start from the proposition that, other things equal, the earnings-price ratio (or its reciprocal, the times-earnings multiplier) of a firm's common stock will normally be only slightly affected by "moderate" amounts of debt in the firm's capital structure.[24] Translated into our no-

[22] In Figure 1 the measure of leverage used is D_j/V_j (the ratio of debt to market value) rather than D_j/S_j (the ratio of debt to equity), the concept used in the analytical development. The D_j/V_j measure is introduced at this point because it simplifies comparison and contrast of our view with the traditional position.

[23] The line MM' in Figure 2 has been drawn with a positive slope on the assumption that $\rho_k^\tau > r$, a condition which will normally obtain. Our Proposition II as given in equation (8) would continue to be valid, of course, even in the unlikely event that $\rho_k^\tau < r$, but the slope of MM' would be negative.

[24] See, *e.g.*, Graham and Dodd [6, pp. 464–66]. Without doing violence to this position, we can bring out its implications more sharply by ignoring the qualification and treating the yield as a virtual constant over the relevant range. See in this connection the discussion in Durand [3, esp. pp. 225–37] of what he calls the "net income method" of valuation.

tation, it asserts that for any firm j in the class k,

$$\frac{\bar{X}_j^\tau - rD_j}{S_j} \equiv \frac{\bar{\pi}_j^\tau}{S_j} = i_k^*, \text{ a constant for } \frac{D_j}{S_j} \leq L_k \tag{13}$$

or, equivalently,

$$S_j = \bar{\pi}_j^\tau / i_k^*. \tag{14}$$

Here i_k^* represents the capitalization rate or earnings-price ratio on the common stock and L_k denotes some amount of leverage regarded as the maximum "reasonable" amount for firms of the class k. This assumed relationship between yield and leverage is the horizontal solid line ML' of Figure 2. Beyond L', the yield will presumably rise sharply as the market discounts "excessive" trading on the equity. This possibility of a rising range for high leverages is indicated by the broken-line segment $L'G$ in the figure.[25]

If the value of shares were really given by (14) then the over-all market value of the firm must be:

$$V_j \equiv S_j + D_j = \frac{\bar{X}_j^\tau - rD_j}{i_k^*} + D_j = \frac{\bar{X}_j^\tau}{i_k^*} + \frac{(i_k^* - r)D_j}{i_k^*}. \tag{16}$$

That is, for any given level of expected total returns after taxes ($\bar{X}_j^\tau$) and assuming, as seems natural, that $i_k^* > r$, the value of the firm must tend to *rise* with debt;[26] whereas our Proposition I asserts that the value of the firm is completely independent of the capital structure. Another way of contrasting our position with the traditional one is in terms of the cost of capital. Solving (16) for $\bar{X}_j^\tau / V_j$ yields:

$$\bar{X}_j^\tau / V_j = i_k^* - (i_k^* - r) D_j / V_j. \tag{17}$$

According to this equation, the average cost of capital is not independent of capital structure as we have argued, but should tend to *fall* with increasing leverage, at least within the relevant range of moderate debt ratios, as shown by the line *ms* in Figure 1. Or to put it in more familiar terms, debt-financing should be "cheaper" than equity-financing if not carried too far.

When we also allow for the possibility of a rising range of stock yields for large values of leverage, we obtain a U-shaped curve like *mst* in

[25] To make it easier to see some of the implications of this hypothesis as well as to prepare the ground for later statistical testing, it will be helpful to assume that the notion of a critical limit on leverage beyond which yields rise rapidly, can be epitomized by a quadratic relation of the form:

$$\bar{\pi}_j^\tau / S_j = i_k^* + \beta(D_j/S_j) + \alpha(D_j/S_j)^2, \qquad \alpha > 0. \tag{15}$$

[26] For a typical discussion of how a promoter can, supposedly, increase the market value of a firm by recourse to debt issues, see W. J. Eiteman [4, esp. pp. 11–13].

Figure 1.[27] That a yield-curve for stocks of the form *ML'G* in Figure 2 implies a U-shaped cost-of-capital curve has, of course, been recognized by many writers. A natural further step has been to suggest that the capital structure corresponding to the trough of the U is an "optimal capital structure" towards which management ought to strive in the best interests of the stockholders.[28] According to our model, by contrast, no such optimal structure exists—all structures being equivalent from the point of view of the cost of capital.

Although the falling, or at least U-shaped, cost-of-capital function is in one form or another the dominant view in the literature, the ultimate rationale of that view is by no means clear. The crucial element in the position—that the expected earnings-price ratio of the stock is largely unaffected by leverage up to some conventional limit—is rarely even regarded as something which requires explanation. It is usually simply taken for granted or it is merely asserted that this is the way the market behaves.[29] To the extent that the constant earnings-price ratio has a rationale at all we suspect that it reflects in most cases the feeling that moderate amounts of debt in "sound" corporations do not really add very much to the "riskiness" of the stock. Since the extra risk is slight, it seems natural to suppose that firms will not have to pay noticeably higher yields in order to induce investors to hold the stock.[30]

A more sophisticated line of argument has been advanced by David Durand [3, pp. 231–33]. He suggests that because insurance companies and certain other important institutional investors are restricted to debt securities, nonfinancial corporations are able to borrow from them at interest rates which are lower than would be required to compensate

[27] The U-shaped nature of the cost-of-capital curve can be exhibited explicitly if the yield curve for shares as a function of leverage can be approximated by equation (15) of footnote 25. From that equation, multiplying both sides by S_j we obtain: $\bar{\pi}_j^{\tau} = \bar{X}_j^{\tau} - rD_j = i_k^* S_j + \beta D_j + \alpha D_j^2 / S_j$ or, adding and subtracting $i_k^* D_k$ from the right-hand side and collecting terms,

$$\bar{X}_j^{\tau} = i_k^*(S_j + D_j) + (\beta + r - i_k^*)D_j + \alpha D^2{}_j/S_j. \tag{18}$$

Dividing (18) by V_j gives an expression for the cost of capital:

$$\begin{aligned} \bar{X}_j^{\tau}/V_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha D_j^2/S_jV_j = i_k^* - (i_k^* - r - \beta)D_j/V_j \\ + \alpha(D_j/V_j)^2/(1 - D_j/V_j) \end{aligned} \tag{19}$$

which is clearly U-shaped since α is supposed to be positive.

[28] For a typical statement see S. M. Robbins [16, p. 307]. See also Graham and Dodd [6, pp. 468–74].

[29] See *e.g.*, Graham and Dodd [6, p. 466].

[30] A typical statement is the following by Guthmann and Dougall [7, p. 245]: "Theoretically it might be argued that the increased hazard from using bonds and preferred stocks would counterbalance this additional income and so prevent the common stock from being more attractive than when it had a lower return but fewer prior obligations. In practice, the extra earnings from 'trading on the equity' are often regarded by investors as more than sufficient to serve as a 'premium for risk' when the proportions of the several securities are judiciously mixed."

creditors in a free market. Thus, while he would presumably agree with our conclusions that stockholders could not gain from leverage in an unconstrained market, he concludes that they can gain under present institutional arrangements. This gain would arise by virtue of the "safety superpremium" which lenders are willing to pay corporations for the privilege of lending.[31]

The defective link in both the traditional and the Durand version of the argument lies in the confusion between investors' subjective risk preferences and their objective market opportunities. Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle).

An analogy may he helpful at this point. The relations between $1/\rho_k$, the price per dollar of an unlevered stream in class k; $1/r$, the price per dollar of a sure stream, and $1/i_j$, the price per dollar of a levered stream j, in the kth class, are essentially the same as those between, respectively, the price of whole milk, the price of butter fat, and the price of milk which has been thinned out by skimming off some of the butter fat. Our Proposition I states that a firm cannot reduce the cost of capital—*i.e.*, increase the market value of the stream it generates—by securing part of its capital through the sale of bonds, even though debt money appears to be cheaper. This assertion is equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming the milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk. Similarly our Proposition II—that the price per dollar of a levered stream falls as leverage increases—is an ex-

[31] Like Durand, Morton [15] contends "that the actual market deviates from [Proposition I] by giving a changing over-all cost of money at different points of the [leverage] scale" (p. 443, note 2, inserts ours), but the basis for this contention is nowhere clearly stated. Judging by the great emphasis given to the lack of mobility of investment funds between stocks and bonds and to the psychological and institutional pressures toward debt portfolios (see pp. 444–51 and especially his discussion of the optimal capital structure on p. 453) he would seem to be taking a position very similar to that of Durand above.

act analogue of the statement that the price per gallon of thinned milk falls continuously as more butter fat is skimmed off.[32]

It is clear that this last assertion is true as long as butter fat is worth more per unit weight than whole milk, and it holds even if, for many consumers, taking a little cream out of the milk (adding a little leverage to the stock) does not detract noticeably from the taste (does not add noticeably to the risk). Furthermore the argument remains valid even in the face of instituional limitations of the type envisaged by Durand. For suppose that a large fraction of the population habitually dines in restaurants which are required by law to serve only cream in lieu of milk (entrust their savings to institutional investors who can only buy bonds). To be sure the price of butter fat will then tend to be higher in relation to that of skimmed milk than in the absence such restrictions (the rate of interest will tend to be lower), and this will benefit people who eat at home and who like skim milk (who manage their own portfolio and are able and willing to take risk). But it will still be the case that a farmer cannot gain by skimming some of the butter fat and selling it separately (firm cannot reduce the cost of capital by recourse to borrowed funds).[33]

Our propositions can be regarded as the extension of the classical theory of markets to the particular case of the capital markets. Those who hold the current view—whether they realize it or not—must as-

[32] Let M denote the quantity of whole milk, B/M the proportion of butter fat in the whole milk, and let p_M, p_B and p_α denote, respectively, the price per unit weight of whole milk, butter fat and thinned milk from which a fraction α of the butter fat has been skimmed off. We then have the fundamental perfect market relation:

$$\text{(a)} \qquad p_\alpha(M-\alpha B)+p_B\alpha B = p_M M, \qquad 0 \leq \alpha \leq 1,$$

stating that total receipts will be the same amount $p_M M$, independently of the amount αB of butter fat that may have been sold separately. Since p_M corresponds to $1/\rho$, p_B to $1/r$, p_α to $1/i$, M to $\bar{X}$ and αB to rD, (a) is equivalent to Proposition I, $S+D=\bar{X}/\rho$. From (a) we derive:

$$\text{(b)} \qquad p_\alpha = p_M \frac{M}{M-\alpha B} - p_B \frac{\alpha B}{M-\alpha B}$$

which gives the price of thinned milk as an explicit function of the proportion of butter fat skimmed off; the function decreasing as long as $p_B > p_M$. From (a) also follows:

$$\text{(c)} \qquad 1/p_\alpha = 1/p_M + (1/p_M - 1/p_B)\frac{p_B\alpha B}{p_\alpha(M-\alpha B)}$$

which is the exact analogue of Proposition II, as given by (8).

[33] The reader who likes parables will find that the analogy with interrelated commodity markets can be pushed a good deal farther than we have done in the text. For instance, the effect of changes in the market rate of interest on the over-all cost of capital is the same as the effect of a change in the price of butter on the price of whole milk. Similarly, just as the relation between the prices of skim milk and butter fat influences the kind of cows that will be reared, so the relation between i and r influences the kind of ventures that will be undertaken. If people like butter we shall have Guernseys; if they are willing to pay a high price for safety, this will encourage ventures which promise smaller but less uncertain streams per dollar of physical assets.

sume not merely that there are lags and frictions in the equilibrating process—a feeling we certainly share,[34] claiming for our propositions only that they describe the central tendency around which observations will scatter—but also that there are large and *systematic* imperfections in the market which permanently bias the outcome. This is an assumption that economists, at any rate, will instinctively eye with some skepticism.

In any event, whether such prolonged, systematic departures from equilibrium really exist or whether our propositions are better descriptions of long-run market behavior can be settled only by empirical research. Before going on to the theory of investment it may be helpful, therefore, to look at the evidence.

E. *Some Preliminary Evidence on the Basic Propositions*

Unfortunately the evidence which has been assembled so far is amazingly skimpy. Indeed, we have been able to locate only two recent studies—and these of rather limited scope—which were designed to throw light on the issue. Pending the results of more comprehensive tests which we hope will soon be available, we shall review briefly such evidence as is provided by the two studies in question: (1) an analysis of the relation between security yields and financial structure for some 43 large electric utilities by F. B. Allen [1], and (2) a parallel (unpublished) study by Robert Smith [19], for 42 oil companies designed to test whether Allen's rather striking results would be found in an industry with very different characteristics.[35] The Allen study is based on average figures for the years 1947 and 1948, while the Smith study relates to the single year 1953.

The Effect of Leverage on the Cost of Capital. According to the received view, as shown in equation (17) the average cost of capital, $\overline{X}^\tau/V$, should decline linearly with leverage as measured by the ratio D/V, at least through most of the relevant range.[36] According to Proposition I, the average cost of capital within a given class k should tend to have the same value ρ_k^τ independently of the degree of leverage. A simple test

[34] Several specific examples of the failure of the arbitrage mechanism can be found in Graham and Dodd [6, *e.g.*, pp. 646–48]. The price discrepancy described on pp. 646–47 is particularly curious since it persists even today despite the fact that a whole generation of security analysts has been brought up on this book!

[35] We wish to express our thanks to both writers for making available to us some of their original worksheets. In addition to these recent studies there is a frequently cited (but apparently seldom read) study by the Federal Communications Commission in 1938 [22] which purports to show the existence of an optimal capital structure or range of structures (in the sense defined above) for public utilities in the 1930's. By current standards for statistical investigations, however, this study cannot be regarded as having any real evidential value for the problem at hand.

[36] We shall simplify our notation in this section by dropping the subscript j used to denote a particular firm wherever this will not lead to confusion.

of the merits of the two alternative hypotheses can thus be carried out by correlating $\bar{X}^\tau/V$ with D/V. If the traditional view is correct, the correlation should be significantly negative; if our view represents a better approximation to reality, then the correlation should not be significantly different from zero.

Both studies provide information about the average value of D—the market value of bonds and preferred stock—and of V—the market value of all securities.[37] From these data we can readily compute the ratio D/V and this ratio (expressed as a percentage) is represented by the symbol d in the regression equations below. The measurement of the variable $\bar{X}^\tau/V$, however, presents serious difficulties. Strictly speaking, the numerator should measure the expected returns net of taxes, but this is a variable on which no direct information is available. As an approximation, we have followed both authors and used (1) the average value of actual net returns in 1947 and 1948 for Allen's utilities; and (2) actual net returns in 1953 for Smith's oil companies. Net return is defined in both cases as the sum of interest, preferred dividends and stockholders' income net of corporate income taxes. Although this approximation to expected returns is undoubtedly very crude, there is no reason to believe that it will systematically bias the test in so far as the sign of the regression coefficient is concerned. The roughness of the approximation, however, will tend to make for a wide scatter. Also contributing to the scatter is the crudeness of the industrial classification, since especially within the sample of oil companies, the assumption that all the firms belong to the same class in our sense, is at best only approximately valid.

Denoting by x our approximation to $\bar{X}^\tau/V$ (expressed, like d, as a percentage), the results of the tests are as follows:

$$\text{Electric Utilities} \quad x = 5.3 + \underset{(\pm .008)}{.006d} \qquad r = .12$$

$$\text{Oil Companies} \quad x = 8.5 + \underset{(\pm .024)}{.006d} \qquad r = .04.$$

The data underlying these equations are also shown in scatter diagram form in Figures 3 and 4.

The results of these tests are clearly favorable to our hypothesis.

[37] Note that for purposes of this test preferred stocks, since they represent an *expected* fixed obligation, are properly classified with bonds even though the tax status of preferred dividends is different from that of interest payments and even though preferred dividends are really fixed only as to their maximum in any year. Some difficulty of classification does arise in the case of convertible preferred stocks (and convertible bonds) selling at a substantial premium, but fortunately very few such issues were involved for the companies included in the two studies. Smith included bank loans and certain other short-term obligations (at book values) in his data on oil company debts and this treatment is perhaps open to some question. However, the amounts involved were relatively small and check computations showed that their elimination would lead to only minor differences in the test results.

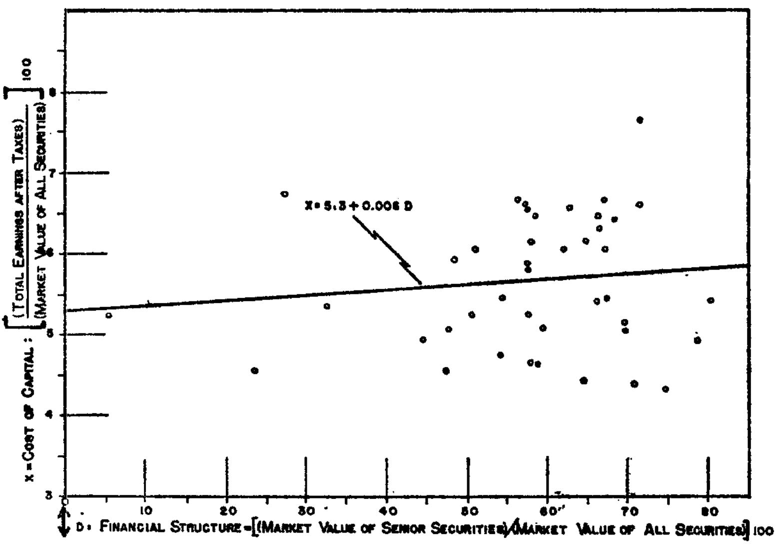


FIGURE 3. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 43 ELECTRIC UTILITIES, 1947–48

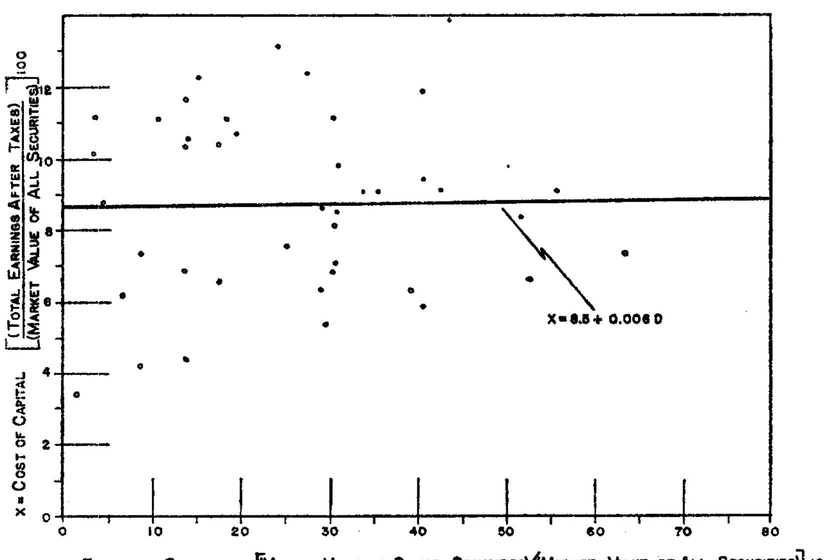


FIGURE 4. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 42 OIL COMPANIES, 1953

Both correlation coefficients are very close to zero and not statistically significant. Furthermore, the implications of the traditional view fail to be supported even with respect to the sign of the correlation. The data in short provide no evidence of any tendency for the cost of capital to fall as the debt ratio increases.[38]

It should also be apparent from the scatter diagrams that there is no hint of a curvilinear, U-shaped, relation of the kind which is widely believed to hold between the cost of capital and leverage. This graphical impression was confirmed by statistical tests which showed that for both industries the curvature was not significantly different from zero, its sign actually being opposite to that hypothesized.[39]

Note also that according to our model, the constant terms of the regression equations are measures of ρ_k^τ, the capitalization rates for unlevered streams and hence the average cost of capital in the classes in question. The estimates of 8.5 per cent for the oil companies as against 5.3 per cent for electric utilities appear to accord well with a priori expectations, both in absolute value and relative spread.

The Effect of Leverage on Common Stock Yields. According to our Proposition II—see equation 12 and Figure 2—the expected yield on common stock, $\bar{\pi}^\tau/S$, in any given class, should tend to increase with leverage as measured by the ratio D/S. The relation should tend to be linear and with positive slope through most of the relevant range (as in the curve MM' of Figure 2), though it might tend to flatten out if we move

[38] It may be argued that a test of the kind used is biased against the traditional view. The fact that both sides of the regression equation are divided by the variable V which may be subject to random variation might tend to impart a positive bias to the correlation. As a check on the results presented in the text, we have, therefore, carried out a supplementary test based on equation (16). This equation shows that, if the traditional view is correct, the market value of a company should, for given $\bar{X}^\tau$, increase with debt through most of the relevant range; according to our model the market value should be uncorrelated with D, given $\bar{X}^\tau$. Because of wide variations in the size of the firms included in our samples, all variables must be divided by a suitable scale factor in order to avoid spurious results in carrying out a test of equation (16). The factor we have used is the book value of the firm denoted by A. The hypothesis tested thus takes the specific form:

$$V/A = a + b(\bar{X}^\tau/A) + c(D/A)$$

and the numerator of the ratio X^τ/A is again approximated by actual net returns. The partial correlation between V/A and D/A should now be positive according to the traditional view and zero according to our model. Although division by A should, if anything, bias the results in favor of the traditional hypothesis, the partial correlation turns out to be only .03 for the oil companies and −.28 for the electric utilities. Neither of these coefficients is significantly different from zero and the larger one even has the wrong sign.

[39] The tests consisted of fitting to the data the equation (19) of footnote 27. As shown there, it follows from the U-shaped hypothesis that the coefficient α of the variable $(D/V)^2/(1-D/V)$, denoted hereafter by d^*, should be significant and positive. The following regression equations and partials were obtained:

$$\text{Electric Utilities} \quad x = 5.0 + .017d - .003d^*; \; r_{xd^*.d} = -.15$$

$$\text{Oil Companies} \quad x = 8.0 + .05d - .03d^*; \; r_{xd^*.d} = -.14.$$

far enough to the right (as in the curve MD'), to the extent that high leverage tends to drive up the cost of senior capital. According to the conventional view, the yield curve as a function of leverage should be a horizontal straight line (like ML') through most of the relevant range; far enough to the right, the yield may tend to rise at an increasing rate. Here again, a straight-forward correlation—in this case between $\bar{\pi}^\tau/S$ and D/S—can provide a test of the two positions. If our view is correct, the correlation should be significantly positive; if the traditional view is correct, the correlation should be negligible.

Subject to the same qualifications noted above in connection with $\bar{X}^\tau$, we can approximate $\bar{\pi}^\tau$ by actual stockholder net income.[40] Letting z denote in each case the approximation to $\bar{\pi}^\tau/S$ (expressed as a percentage) and letting h denote the ratio D/S (also in percentage terms) the following results are obtained:

$$\text{Electric Utilities} \quad z = \underset{(+.004)}{6.6 + .017h} \qquad r = .53$$

$$\text{Oil Companies} \quad z = \underset{(\pm .012)}{8.9 + .051h} \qquad r = .53.$$

These results are shown in scatter diagram form in Figures 5 and 6.

Here again the implications of our analysis seem to be borne out by the data. Both correlation coefficients are positive and highly significant when account is taken of the substantial sample size. Furthermore, the estimates of the coefficients of the equations seem to accord reasonably well with our hypothesis. According to equation (12) the constant term should be the value of ρ_k^τ for the given class while the slope should be $(\rho_k^\tau - r)$. From the test of Proposition I we have seen that for the oil companies the mean value of ρ_k^τ could be estimated at around 8.7. Since the average yield of senior capital during the period covered was in the order of $3\frac{1}{2}$ per cent, we should expect a constant term of about 8.7 per cent and a slope of just over 5 per cent. These values closely approximate the regression estimates of 8.9 per cent and 5.1 per cent respectively. For the electric utilities, the yield of senior capital was also on the order of $3\frac{1}{2}$ per cent during the test years, but since the estimate of the mean value of ρ_k^τ from the test of Proposition I was 5.6 per cent,

[40] As indicated earlier, Smith's data were for the single year 1953. Since the use of a single year's profits as a measure of expected profits might be open to objection we collected profit data for 1952 for the same companies and based the computation of $\bar{\pi}^\tau/S$ on the average of the two years. The value of $\bar{\pi}^\tau/S$ was obtained from the formula:

$$\left(\text{net earnings in 1952} \cdot \frac{\text{assets in '53}}{\text{assets in '52}} + \text{net earnings in '1953}\right)\frac{1}{2}$$
$$\div \text{ (average market value of common stock in '53).}$$

The asset adjustment was introduced as rough allowance for the effects of possible growth in the size of the firm. It might be added that the correlation computed with $\bar{\pi}^\tau/S$ based on net profits in 1953 alone was found to be only slightly smaller, namely .50.

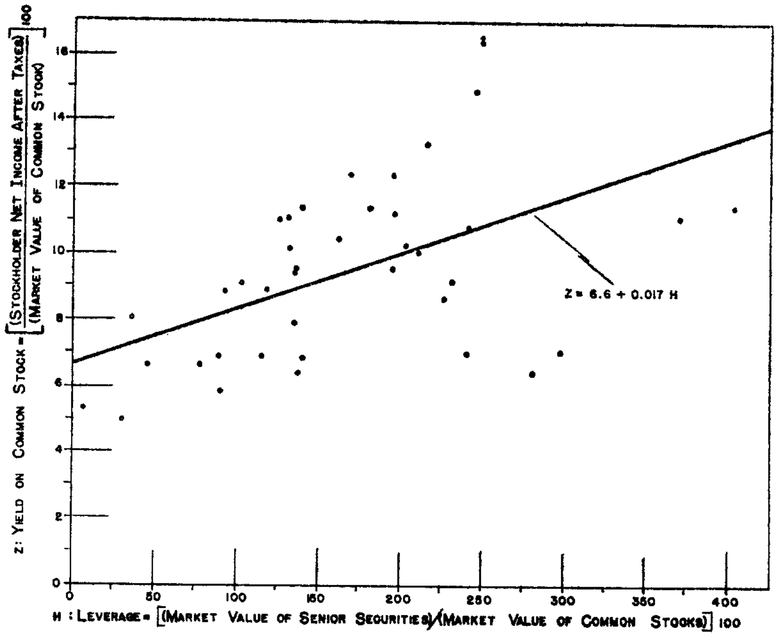


Figure 5. Yield on Common Stock in Relation to Leverage for 43 Electric Utilities, 1947–48

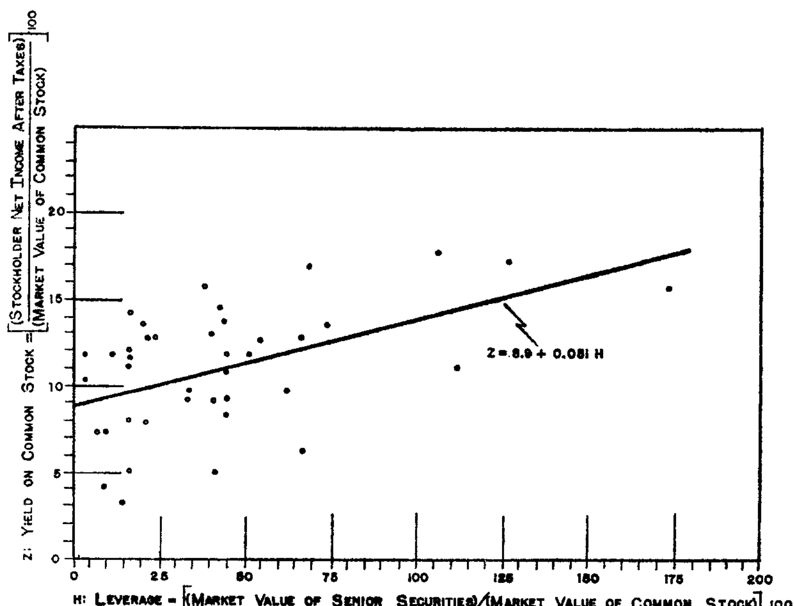


Figure 6. Yield on Common Stock in Relation to Leverage for 42 Oil Companies, 1952–53

the slope should be just above 2 per cent. The actual regression estimate for the slope of 1.7 per cent is thus somewhat low, but still within one standard error of its theoretical value. Because of this underestimate of the slope and because of the large mean value of leverage ($\bar{h}=160$ per cent) the regression estimate of the constant term, 6.6 per cent, is somewhat high, although not significantly different from the value of 5.6 per cent obtained in the test of Proposition I.

When we add a square term to the above equations to test for the presence and direction of curvature we obtain the following estimates:

$$\text{Electric Utilities} \quad z = 4.6 + .004h - .007h^2$$

$$\text{Oil Companies} \quad z = 8.5 + .072h - .016h^2.$$

For both cases the curvature is negative. In fact, for the electric utilities, where the observations cover a wider range of leverage ratios, the negative coefficient of the square term is actually significant at the 5 per cent level. Negative curvature, as we have seen, runs directly counter to the traditional hypothesis, whereas it can be readily accounted for by our model in terms of rising cost of borrowed funds.[41]

In summary, the empirical evidence we have reviewed seems to be broadly consistent with our model and largely inconsistent with traditional views. Needless to say much more extensive testing will be required before we can firmly conclude that our theory describes market behavior. Caution is indicated especially with regard to our test of Proposition II, partly because of possible statistical pitfalls[42] and partly because not all the factors that might have a systematic effect on stock yields have been considered. In particular, no attempt was made to test the possible influence of the dividend pay-out ratio whose role has tended to receive a great deal of attention in current research and thinking. There are two reasons for this omission. First, our main objective has been to assess the prima facie tenability of *our* model, and in this model, based as it is on rational behavior by investors, dividends per se play no role. Second, in a world in which the policy of dividend stabilization is widespread, there is no simple way of disentangling the true effect of dividend payments on stock prices from their apparent effect,

[41] That the yield of senior capital tended to rise for utilities as leverage increased is clearly shown in several of the scatter diagrams presented in the published version of Allen's study. This significant negative curvature between stock yields and leverage for utilities may be partly responsible for the fact, previously noted, that the constant in the linear regression is somewhat higher and the slope somewhat lower than implied by equation (12). Note also in connection with the estimate of ρ_k^τ that the introduction of the quadratic term reduces the constant considerably, pushing it in fact below the a priori expectation of 5.6, though the difference is again not statistically significant.

[42] In our test, *e.g.*, the two variables z and h are both ratios with S appearing in the denominator, which may tend to impart a positive bias to the correlation (*cf.* note 38). Attempts were made to develop alternative tests, but although various possibilities were explored, we have so far been unable to find satisfactory alternatives.

the latter reflecting only the role of dividends as a proxy measure of long-term earning anticipations.[43] The difficulties just mentioned are further compounded by possible interrelations between dividend policy and leverage.[44]

II. *Implications of the Analysis for the Theory of Investment*

A. *Capital Structure and Investment Policy*

On the basis of our propositions with respect to cost of capital and financial structure (and for the moment neglecting taxes), we can derive the following simple rule for optimal investment policy by the firm:

Proposition III. If a firm in class k is acting in the best interest of the stockholders at the time of the decision, it will exploit an investment opportunity if and only if the rate of return on the investment, say ρ^*, is as large as or larger than ρ_k. That is, *the cut-off point for investment in the firm will in all cases be ρ_k and will be completely unaffected by the type of security used to finance the investment.* Equivalently, we may say that regardless of the financing used, the marginal cost of capital to a firm is equal to the average cost of capital, which is in turn equal to the capitalization rate for an unlevered stream in the class to which the firm belongs.[45]

To establish this result we will consider the three major financing alternatives open to the firm—bonds, retained earnings, and common stock issues—and show that in each case an investment is worth undertaking if, and only if, $\rho^* \geqq \rho_k$.[46]

Consider first the case of an investment financed by the sale of bonds. We know from Proposition I that the market value of the firm before the investment was undertaken was:[47]

$$V_0 = \overline{X}_0/\rho_k \tag{20}$$

[43] We suggest that failure to appreciate this difficulty is responsible for many fallacious, or at least unwarranted, conclusions about the role of dividends.

[44] In the sample of electric utilities, there is a substantial negative correlation between yields and pay-out ratios, but also between pay-out ratios and leverage, suggesting that either the association of yields and leverage or of yields and pay-out ratios may be (at least partly) spurious. These difficulties however do not arise in the case of the oil industry sample. A preliminary analysis indicates that there is here no significant relation between leverage and pay-out ratios and also no significant correlation (either gross or partial) between yields and pay-out ratios.

[45] The analysis developed in this paper is essentially a comparative-statics, not a dynamic analysis. This note of caution applies with special force to Proposition III. Such problems as those posed by expected changes in r and in ρ_k over time will not be treated here. Although they are in principle amenable to analysis within the general framework we have laid out, such an undertaking is sufficiently complex to deserve separate treatment. *Cf.* note 17.

[46] The extension of the proof to other types of financing, such as the sale of preferred stock or the issuance of stock rights is straightforward.

[47] Since no confusion is likely to arise, we have again, for simplicity, eliminated the subscripts identifying the firm in the equations to follow. Except for ρ_k, the subscripts now refer to time periods.

and that the value of the common stock was:

$$S_0 = V_0 - D_0. \tag{21}$$

If now the firm borrows I dollars to finance an investment yielding ρ^* its market value will become:

$$V_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} = V_0 + \frac{\rho^* I}{\rho_k} \tag{22}$$

and the value of its common stock will be:

$$S_1 = V_1 - (D_0 + I) = V_0 + \frac{\rho^* I}{\rho_k} - D_0 - I \tag{23}$$

or using equation 21,

$$S_1 = S_0 + \frac{\rho^* I}{\rho_k} - I. \tag{24}$$

Hence $S_1 \gtreqless S_0$ as $\rho^* \gtreqless \rho_k$.[48]

To illustrate, suppose the capitalization rate for uncertain streams in the kth class is 10 per cent and the rate of interest is 4 per cent. Then if a given company had an expected income of 1,000 and if it were financed entirely by common stock we know from Proposition I that the market value of its stock would be 10,000. Assume now that the managers of the firm discover an investment opportunity which will require an outlay of 100 and which is expected to yield 8 per cent. At first sight this might appear to be a profitable opportunity since the expected return is double the interest cost. If, however, the management borrows the necessary 100 at 4 per cent, the total expected income of the company rises to 1,008 and the market value of the firm to 10,080. But the firm now will have 100 of bonds in its capital structure so that, paradoxically, the market value of the stock must actually be reduced from 10,000 to 9,980 as a consequence of this apparently profitable investment. Or, to put it another way, the gains from being able to tap cheap, borrowed funds are more than offset for the stockholders by the market's discounting of the stock for the added leverage assumed.

Consider next the case of retained earnings. Suppose that in the course of its operations the firm acquired I dollars of cash (without impairing

[48] In the case of bond-financing the rate of interest on bonds does not enter explicitly into the decision (assuming the firm borrows at the market rate of interest). This is true, moreover, given the conditions outlined in Section I.C, even though interest rates may be an increasing function of debt outstanding. To the extent that the firm borrowed at a rate other than the market rate the two I's in equation (24) would no longer be identical and an additional gain or loss, as the case might be, would accrue to the shareholders. It might also be noted in passing that permitting the two I's in (24) to take on different values provides a simple method for introducing underwriting expenses into the analysis.

the earning power of its assets). If the cash is distributed as a dividend to the stockholders their wealth W_0, after the distribution will be:

$$W_0 = S_0 + I = \frac{\overline{X}_0}{\rho_k} - D_0 + I \tag{25}$$

where $\overline{X}_0$ represents the expected return from the assets exclusive of the amount I in question. If however the funds are retained by the company and used to finance new assets whose expected rate of return is ρ^*, then the stockholders' wealth would become:

$$W_1 = S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k}\,. \tag{26}$$

Clearly $W_1 \gtreqless W_0$ as $\rho^* \gtreqless \rho_k$ so that an investment financed by retained earnings raises the net worth of the owners if and only if $\rho^* > \rho_k$.[49]

Consider finally, the case of common-stock financing. Let P_0 denote the current market price per share of stock and assume, for simplicity, that this price reflects currently expected earnings only, that is, it does not reflect any future increase in earnings as a result of the investment under consideration.[50] Then if N is the original number of shares, the price per share is:

$$P_0 = S_0/N \tag{27}$$

and the number of new shares, M, needed to finance an investment of I dollars is given by:

$$M = \frac{I}{P_0}\,. \tag{28}$$

As a result of the investment the market value of the stock becomes:

$$S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} = NP_0 + \frac{\rho^* I}{\rho_k}$$

and the price per share:

$$P_1 = \frac{S_1}{N+M} = \frac{1}{N+M}\left[NP_0 + \frac{\rho^* I}{\rho_k}\right]. \tag{29}$$

[49] The conclusion that ρ_k is the cut-off point for investments financed from internal funds applies not only to undistributed net profits, but to depreciation allowances (and even to the funds represented by the current sale value of any asset or collection of assets). Since the owners can earn ρ_k by investing funds elsewhere in the class, partial or total liquidating distributions should be made whenever the firm cannot achieve a marginal internal rate of return equal to ρ_k.

[50] If we assumed that the market price of the stock did reflect the expected higher future earnings (as would be the case if our original set of assumptions above were strictly followed) the analysis would differ slightly in detail, but not in essentials. The cut-off point for new investment would still be ρ_k, but where $\rho^* > \rho_k$ the gain to the original owners would be larger than if the stock price were based on the pre-investment expectations only.

Since by equation (28), $I = MP_0$, we can add MP_0 and subtract I from the quantity in bracket, obtaining:

$$
\begin{aligned}
P_1 &= \frac{1}{N+M}\left[(N+M)P_0 + \frac{\rho^* - \rho_k}{\rho_k} I\right] \\
&= P_0 + \frac{1}{N+M} \frac{\rho^* - \rho_k}{\rho_k} I > P_0 \text{ if,}
\end{aligned} \tag{30}
$$

and only if, $\rho^* > \rho_k$.

Thus an investment financed by common stock is advantageous to the current stockholders if and only if its yield exceeds the capitalization rate ρ_k.

Once again a numerical example may help to illustrate the result and make it clear why the relevant cut-off rate is ρ_k and not the current yield on common stock, i. Suppose that ρ_k is 10 per cent, r is 4 per cent, that the original expected income of our company is 1,000 and that management has the opportunity of investing 100 having an expected yield of 12 per cent. If the original capital structure is 50 per cent debt and 50 per cent equity, and 1,000 shares of stock are initially outstanding, then, by Proposition I, the market value of the common stock must be 5,000 or 5 per share. Furthermore, since the interest bill is $.04 \times 5{,}000 = 200$, the yield on common stock is $800/5{,}000 = 16$ per cent. It may then appear that financing the additional investment of 100 by issuing 20 shares to outsiders at 5 per share would dilute the equity of the original owners since the 100 promises to yield 12 per cent whereas the common stock is currently yielding 16 per cent. Actually, however, the income of the company would rise to 1,012; the value of the firm to 10,120; and the value of the common stock to 5,120. Since there are now 1,020 shares, each would be worth 5.02 and the wealth of the original stockholders would thus have been increased. What has happened is that the dilution in expected earnings per share (from .80 to .796) has been more than offset, in its effect upon the market price of the shares, by the decrease in leverage.

Our conclusion is, once again, at variance with conventional views,[51] so much so as to be easily misinterpreted. Read hastily, Proposition III seems to imply that the capital structure of a firm is a matter of indifference; and that, consequently, one of the core problems of corporate finance—the problem of the optimal capital structure for a firm—is no problem at all. It may be helpful, therefore, to clear up such possible misundertandings.

[51] In the matter of investment policy under uncertainty there is no single position which represents "accepted" doctrine. For a sample of current formulations, all very different from ours, see Joel Dean [2, esp. Ch. 3], M. Gordon and E. Shapiro [5], and Harry Roberts [17].

B. *Proposition III and Financial Planning by Firms*

Misinterpretation of the scope of Proposition III can be avoided by remembering that this Proposition tells us only that the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worth while. This does not mean that the owners (or the managers) have no grounds whatever for preferring one financing plan to another; or that there are no other policy or technical issues in finance at the level of the firm.

That grounds for preferring one type of financial structure to another will still exist within the framework of our model can readily be seen for the case of common-stock financing. In general, except for something like a widely publicized oil-strike, we would expect the market to place very heavy weight on current and recent past earnings in forming expectations as to future returns. Hence, if the owners of a firm discovered a major investment opportunity which they felt would yield much more than ρ_k, they might well prefer not to finance it via common stock at the then ruling price, because this price may fail to capitalize the new venture. A better course would be a pre-emptive issue of stock (and in this connection it should be remembered that stockholders are free to borrow and buy). Another possibility would be to finance the project initially with debt. Once the project had reflected itself in increased actual earnings, the debt could be retired either with an equity issue at much better prices or through retained earnings. Still another possibility along the same lines might be to combine the two steps by means of a convertible debenture or preferred stock, perhaps with a progressively declining conversion rate. Even such a double-stage financing plan may possibly be regarded as yielding too large a share to outsiders since the new stockholders are, in effect, being given an interest in any similar opportunities the firm may discover in the future. If there is a reasonable prospect that even larger opportunities may arise in the near future and if there is some danger that borrowing now would preclude more borrowing later, the owners might find their interests best protected by splitting off the current opportunity into a separate subsidiary with independent financing. Clearly the problems involved in making the crucial estimates and in planning the optimal financial strategy are by no means trivial, even though they should have no bearing on the basic decision to invest (as long as $\rho^* \geqq \rho_k$).[52]

Another reason why the alternatives in financial plans may not be a matter of indifference arises from the fact that managers are concerned

[52] Nor can we rule out the possibility that the existing owners, if unable to use a financing plan which protects their interest, may actually prefer to pass up an otherwise profitable venture rather than give outsiders an "excessive" share of the business. It is presumably in situations of this kind that we could justifiably speak of a shortage of "equity capital," though this kind of market imperfection is likely to be of significance only for small or new firms.

with more than simply furthering the interest of the owners. Such other objectives of the management—which need not be necessarily in conflict with those of the owners—are much more likely to be served by some types of financing arrangements than others. In many forms of borrowing agreements, for example, creditors are able to stipulate terms which the current management may regard as infringing on its prerogatives or restricting its freedom to maneuver. The creditors might even be able to insist on having a direct voice in the formation of policy.[53] To the extent, therefore, that financial policies have these implications for the management of the firm, something like the utility approach described in the introductory section becomes relevant to financial (as opposed to investment) decision-making. It is, however, the utility functions of the managers per se and not of the owners that are now involved.[54]

In summary, many of the specific considerations which bulk so large in traditional discussions of corporate finance can readily be superimposed on our simple framework without forcing any drastic (and certainly no systematic) alteration of the conclusion which is our principal concern, namely that for investment decisions, the marginal cost of capital is ρ_k.

C. *The Effect of the Corporate Income Tax on Investment Decisions*

In Section I it was shown that when an unintegrated corporate income tax is introduced, the original version of our Proposition I,

$$\bar{X}/V = \rho_k = \text{a constant}$$

must be rewritten as:

$$\frac{(\bar{X} - rD)(1 - \tau) + rD}{V} \equiv \frac{\bar{X}^\tau}{V} = \rho_k{}^\tau = \text{a constant.} \tag{11}$$

Throughout Section I we found it convenient to refer to $\bar{X}^\tau/V$ as the cost of capital. The appropriate measure of the cost of capital relevant

[53] Similar considerations are involved in the matter of dividend policy. Even though the stockholders may be indifferent as to payout policy as long as investment policy is optimal, the management need not be so. Retained earnings involve far fewer threats to control than any of the alternative sources of funds and, of course, involve no underwriting expense or risk. But against these advantages management must balance the fact that sharp changes in dividend rates, which heavy reliance on retained earnings might imply, may give the impression that a firm's finances are being poorly managed, with consequent threats to the control and professional standing of the management.

[54] In principle, at least, this introduction of management's risk preferences with respect to financing methods would do much to reconcile the apparent conflict between Proposition III and such empirical findings as those of Modigliani and Zeman [14] on the close relation between interest rates and the ratio of new debt to new equity issues; or of John Lintner [12] on the considerable stability in target and actual dividend-payout ratios.

to investment decisions, however, is the ratio of the expected return *before* taxes to the market value, *i.e.*, $\bar{X}/V$. From (11) above we find:

$$\frac{\bar{X}}{V} = \frac{\rho_k^\tau - \tau_r(D/V)}{1-\tau} = \frac{\rho_k^\tau}{1-\tau}\left[1 - \frac{\tau r D}{\rho_k^\tau V}\right], \tag{31}$$

which shows that the cost of capital now depends on the debt ratio, decreasing, as D/V rises, at the constant rate $\tau r/(1-\tau)$.[55] Thus, with a corporate income tax under which interest is a deductible expense, gains can accrue to stockholders from having debt in the capital structure, even when capital markets are perfect. The gains however are small, as can be seen from (31), and as will be shown more explicitly below.

From (31) we can develop the tax-adjusted counterpart of Proposition III by interpreting the term D/V in that equation as the proportion of debt used in any additional financing of V dollars. For example, in the case where the financing is entirely by new common stock, $D=0$ and the required rate of return ρ_k^S on a venture so financed becomes:

$$\rho_k^S = \frac{\rho_k^\tau}{1-\tau}. \tag{32}$$

For the other extreme of pure debt financing $D=V$ and the required rate of return, ρ_k^D, becomes:

$$\rho_k^D = \frac{\rho_k^\tau}{1-\tau}\left[1 - \tau\frac{r}{\rho_k^\tau}\right] = \rho_k^S\left[1 - \tau\frac{r}{\rho_k^\tau}\right] = \rho_k^S - \frac{\tau}{1-\tau}r.\text{[56]} \tag{33}$$

For investments financed out of retained earnings, the problem of defining the required rate of return is more difficult since it involves a comparison of the tax consequences to the individual stockholder of receiving a dividend versus having a capital gain. Depending on the time of realization, a capital gain produced by retained earnings may be taxed either at ordinary income tax rates, 50 per cent of these rates, 25 per

[55] Equation (31) is amenable, in principle, to statistical tests similar to those described in Section I.E. However we have not made any systematic attempt to carry out such tests so far, because neither the Allen nor the Smith study provides the required information. Actually, Smith's data included a very crude estimate of tax liability, and, using this estimate, we did in fact obtain a negative relation between $\bar{X}/V$ and D/V. However, the correlation (−.28) turned out to be significant only at about the 10 per cent level. While this result is not conclusive, it should be remembered that, according to our theory, the slope of the regression equation should be in any event quite small. In fact, with a value of τ in the order of .5, and values of ρ_k^τ and r in the order of 8.5 and 3.5 per cent respectively (*cf.* Section I.E) an increase in D/V from 0 to 60 per cent (which is, approximately, the range of variation of this variable in the sample) should tend to reduce the average cost of capital only from about 17 to about 15 per cent.

[56] This conclusion does not extend to preferred stocks even though they have been classed with debt issues previously. Since preferred dividends except for a portion of those of public utilities are not in general deductible from the corporate tax, the cut-off point for new financing via preferred stock is exactly the same as that for common stock.

cent, or zero, if held till death. The rate on any dividends received in the event of a distribution will also be a variable depending on the amount of other income received by the stockholder, and with the added complications introduced by the current dividend-credit provisions. If we assume that the managers proceed on the basis of reasonable estimates as to the average values of the relevant tax rates for the owners, then the required return for retained earnings $\rho_k{}^R$ can be shown to be:

$$\rho_k{}^R = \rho_k{}^\tau \frac{1}{1-\tau} \frac{1-\tau_d}{1-\tau_g} = \frac{1-\tau_d}{1-\tau_g} \rho_k{}^* \tag{34}$$

where τ_d is the assumed rate of personal income tax on dividends and τ_g is the assumed rate of tax on capital gains.

A numerical illustration may perhaps be helpful in clarifying the relationship between these required rates of return. If we take the following round numbers as representative order-of-magnitude values under present conditions: an after-tax capitalization rate $\rho_k{}^\tau$ of 10 per cent, a rate of interest on bonds of 4 per cent, a corporate tax rate of 50 per cent, a marginal personal income tax rate on dividends of 40 per cent (corresponding to an income of about $25,000 on a joint return), and a capital gains rate of 20 per cent (one-half the marginal rate on dividends), then the required rates of return would be: (1) 20 per cent for investments financed entirely by issuance of new common shares; (2) 16 per cent for investments financed entirely by new debt; and (3) 15 per cent for investments financed wholly from internal funds.

These results would seem to have considerable significance for current discussions of the effect of the corporate income tax on financial policy and on investment. Although we cannot explore the implications of the results in any detail here, we should at least like to call attention to the remarkably small difference between the "cost" of equity funds and debt funds. With the numerical values assumed, equity money turned out to be only 25 per cent more expensive than debt money, rather than something on the order of 5 times as expensive as is commonly supposed to be the case.[57] The reason for the wide difference is that the traditional

[57] See *e.g.*, D. T. Smith [18]. It should also be pointed out that our tax system acts in other ways to reduce the gains from debt financing. Heavy reliance on debt in the capital structure, for example, commits a company to paying out a substantial proportion of its income in the form of interest payments taxable to the owners under the personal income tax. A debt-free company, by contrast, can reinvest in the business all of its (smaller) net income and to this extent subject the owners only to the low capital gains rate (or possibly no tax at all by virtue of the loophole at death). Thus, we should expect a high degree of leverage to be of value to the owners, even in the case of closely held corporations, primarily in cases where their firm was not expected to have much need for additional funds to expand assets and earnings in the future. To the extent that opportunities for growth were available, as they presumably would be for most successful corporations, the interest of the stockholders would tend to be better served by a structure which permitted maximum use of retained earnings.

view starts from the position that debt funds are several times cheaper than equity funds even in the absence of taxes, with taxes serving simply to magnify the cost ratio in proportion to the corporate rate. By contrast, in our model in which the repercussions of debt financing on the value of shares are taken into account, the *only* difference in cost is that due to the tax effect, and its magnitude is simply the tax on the "grossed up" interest payment. Not only is this magnitude likely to be small but our analysis yields the further paradoxical implication that the stockholders' gain from, and hence incentive to use, debt financing is actually smaller the lower the rate of interest. In the extreme case where the firm could borrow for practically nothing, the advantage of debt financing would also be practically nothing.

III. *Conclusion*

With the development of Proposition III the main objectives we outlined in our introductory discussion have been reached. We have in our Propositions I and II at least the foundations of a theory of the valuation of firms and shares in a world of uncertainty. We have shown, moreover, how this theory can lead to an operational definition of the cost of capital and how that concept can be used in turn as a basis for rational investment decision-making within the firm. Needless to say, however, much remains to be done before the cost of capital can be put away on the shelf among the solved problems. Our approach has been that of static, partial equilibrium analysis. It has assumed among other things a state of atomistic competition in the capital markets and an ease of access to those markets which only a relatively small (though important) group of firms even come close to possessing. These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share.

References

1. F. B. Allen, "Does Going into Debt Lower the 'Cost of Capital'?," *Analysts Jour.*, Aug. 1954, *10*, 57–61.
2. J. Dean, *Capital Budgeting*. New York 1951.
3. D. Durand, "Costs of Debt and Equity Funds for Business: Trends and Problems of Measurement" in Nat. Bur. Econ. Research, *Conference on Research in Business Finance*. New York 1952, pp. 215–47.
4. W. J. Eiteman, "Financial Aspects of Promotion," in *Essays on Business Finance* by M. W. Waterford and W. J. Eiteman. Ann Arbor, Mich. 1952, pp. 1–17.
5. M. J. Gordon and E. Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," *Manag. Sci.*, Oct. 1956, *3*, 102–10.

6. B. Graham and L. Dodd, *Security Analysis*, 3rd ed. New York 1951.
7. G. Guthmann and H. E. Dougall, *Corporate Financial Policy*, 3rd ed. New York 1955.
8. J. R. Hicks, *Value and Capital*, 2nd ed. Oxford 1946.
9. P. Hunt and M. Williams, *Case Problems in Finance*, rev. ed. Homewood, Ill. 1954.
10. J. M. Keynes, *The General Theory of Employment, Interest and Money*. New York 1936.
11. O. Lange, *Price Flexibility and Employment*. Bloomington, Ind. 1944.
12. J. Lintner, "Distribution of Incomes of Corporations among Dividends, Retained Earnings and Taxes," *Am. Econ. Rev.*, May 1956, *46*, 97–113.
13. F. Lutz and V. Lutz, *The Theory of Investment of the Firm*. Princeton 1951.
14. F. Modigliani and M. Zeman, "The Effect of the Availability of Funds, and the Terms Thereof, on Business Investment" in Nat. Bur. Econ. Research, *Conference on Research in Business Finance*. New York 1952, pp. 263–309.
15. W. A. Morton, "The Structure of the Capital Market and the Price of Money," *Am. Econ. Rev.*, May 1954, *44*, 440–54.
16. S. M. Robbins, *Managing Securities*. Boston 1954.
17. H. V. Roberts, "Current Problems in the Economics of Capital Budgeting," *Jour. Bus.*, 1957, *30* (1), 12–16.
18. D. T. Smith, *Effects of Taxation on Corporate Financial Policy*. Boston 1952.
19. R. Smith, "Cost of Capital in the Oil Industry," (hectograph). Pittsburgh: Carnegie Inst. Tech. 1955.
20. H. M. Somers, " 'Cost of Money' as the Determinant of Public Utility Rates," *Buffalo Law Rev.*, Spring 1955, *4*, 1–28.
21. J. B. Williams, *The Theory of Investment Value*. Cambridge, Mass. 1938.
22. U. S. Federal Communications Commission, *The Problem of the "Rate of Return" in Public Utility Regulation*. Washington 1938.

THE JOURNAL OF FINANCE • VOL. LVI, NO. 1 • FEBRUARY 2001

Explaining the Rate Spread on Corporate Bonds

EDWIN J. ELTON, MARTIN J. GRUBER, DEEPAK AGRAWAL,
and CHRISTOPHER MANN*

ABSTRACT

The purpose of this article is to explain the spread between rates on corporate and government bonds. We show that expected default accounts for a surprisingly small fraction of the premium in corporate rates over treasuries. While state taxes explain a substantial portion of the difference, the remaining portion of the spread is closely related to the factors that we commonly accept as explaining risk premiums for common stocks. Both our time series and cross-sectional tests support the existence of a risk premium on corporate bonds.

THE PURPOSE OF THIS ARTICLE is to examine and explain the differences in the rates offered on corporate bonds and those offered on government bonds (spreads), and, in particular, to examine whether there is a risk premium in corporate bond spreads and, if so, why it exists.

Spreads in rates between corporate and government bonds differ across rating classes and should be positive for each rating class for the following reasons:

1. Expected default loss—some corporate bonds will default and investors require a higher promised payment to compensate for the expected loss from defaults.
2. Tax premium—interest payments on corporate bonds are taxed at the state level whereas interest payments on government bonds are not.
3. Risk premium—The return on corporate bonds is riskier than the return on government bonds, and investors should require a premium for the higher risk. As we will show, this occurs because a large part of the risk on corporate bonds is systematic rather than diversifiable.

The only controversial part of the above analyses is the third point. Some authors in their analyses assume that the risk premium is zero in the corporate bond market.[1]

* Edwin J. Elton and Martin J. Gruber are Nomura Professors of Finance, Stern School of Business, New York University. Deepak Agrawal and Christopher Mann are Doctoral Students, Stern School of Business, New York University. We would like to thank the Editor, René Stulz, and the Associate Editor for helpful comments and suggestions.

[1] Many authors assume a zero risk premium. Bodie, Kane, and Marcus (1993) assume the spread is all default premium. See also Fons (1994) and Cumby and Evans (1995). On the other hand, rating-based pricing models like Jarrow, Lando, and Turnbull (1997) and Das-Tufano (1996) assume that any risk premium impounded in corporate spreads is captured by adjusting transition probabilities.

This paper is important because it provides the reader with explicit estimates of the size of each of the components of the spread between corporate bond rates and government bond rates.[2] Although some studies have examined losses from default, to the best of our knowledge, none of these studies has examined tax effects or made the size of compensation for systematic risk explicit. Tax effects occur because the investor in corporate bonds is subject to state and local taxes on interest payments, whereas government bonds are not subject to these taxes. Thus, corporate bonds have to offer a higher pre-tax return to yield the same after-tax return. This tax effect has been ignored in the empirical literature on corporate bonds. In addition, past research has ignored or failed to measure whether corporate bond prices contain a risk premium above and beyond the expected loss from default (we find that the risk premium is a large part of the spread). We show that corporate bonds require a risk premium because spreads and returns vary systematically with the same factors that affect common stock returns. If investors in common stocks require compensation for this risk, so should investors in corporate bonds. The source of the risk premium in corporate bond prices has long been a puzzle to researchers and this study is the first to provide both an explanation of why it exists and an estimate of its importance.

Why do we care about estimating the spread components separately for various maturities and rating classes rather than simply pricing corporate bonds off a spot yield curve or a set of estimated risk neutral probabilities? First, we want to know the factors affecting the value of assets and not simply their value. Second, for an investor thinking about purchasing a corporate bond, the size of each component for each rating class will affect the decision of whether to purchase a particular class of bonds or whether to purchase corporate bonds at all.

To illustrate this last point, consider the literature that indicates that low-rated bonds produce higher average returns than bonds with higher ratings whereas the lower-rated bonds do not have a higher standard deviation of return.[3] What does this evidence indicate for investment? This evidence has been used to argue that low-rated bonds are attractive investments. However, we know that this is only true if required return is no higher for low-rated debt. Our decomposition of corporate spreads shows that the risk premium increases for lower-rated debt. In addition, because promised coupon is higher for lower-rated debt, the tax burden is greater. Thus, the fact that lower-rated bonds have higher realized returns does not imply they are better investments because the higher realized return might not be sufficient compensation for taxes and risk.

[2] Liquidity may play a role in the risk and pricing of corporate bonds. We, like other studies, abstract from this influence.

[3] See, for example, Altman (1989), Goodman (1989), Blume, Keim, and Patel (1991), and Cornell and Green (1991).

The paper proceed as follows: in the first section we start with a description of our sample. We next discuss both the need for using spot rates (the yield on zero-coupon bonds) to compute spreads and the methodology for estimating them. We examine the size and characteristics of the spreads. As a check on the reasonableness of the spot curves, we estimate, for government and corporate bonds, the ability of our estimated spot rates to price bonds. The next three sections (Sections II–IV) of the paper present the heart of our analysis: the decomposition of rate spreads into that part which is due to expected loss, that part which is due to taxes, and that part which is due to the presence of systematic risk.

In the first of these sections (Sec. II), we model and estimate that part of the corporate spread which is due to expected default loss. If we assume for the moment that there is no risk premium, then we can value corporate bonds under the assumption that investors are risk neutral using expected default losses.[4] This risk neutrality assumption allows us to construct a model and estimate what the corporate spot rate spread would be if it were solely due to expected default losses. We find that the spot rate spread curves estimated by incorporating only the expected default losses are well below the observed spot spread curve and that they do not increase as we move to lower ratings as fast as actual spot spread curves. In fact, expected loss can account for no more than 25 percent of the corporate spot spreads.

In Section III, we examine the impact of both the expected default loss and the tax premium on corporate spot spreads. In particular, we build both expected default loss and taxes into the risk neutral valuation model developed earlier and estimate the corporate spot rates that should be used to discount promised cash payments when both state and local taxes and expected default losses are taken into consideration. We then show that using the best estimate of tax rates, actual corporate spot spreads are still much higher than what taxes and default premiums can together account for.

Section IV presents direct evidence of the existence of a risk premium and demonstrates that this risk premium is compensation for the systematic nature of risk in bond returns. We first relate the time series of that part of the spreads that is not explained by expected loss or taxes to variables that are generally considered systematic priced factors in the literature of financial economics. Then we relate cross-sectional differences in spreads to sensitivities of each spread to these variables. We have already shown that the default premium and tax premium can only partially account for the difference in corporate spreads. In this section we present direct evidence that there is a premium for systematic risk by showing that the majority of the corporate spread, not explained by defaults or taxes, is explained by factor sensitivities and their prices. Further tests suggest that the factor sensitivities are not proxies for changes in expected default risk.

Conclusions are presented in Section V.

[4] We also temporarily ignore the tax disadvantage of corporate bonds relative to government bonds in this section.

I. Corporate Yield Spreads

In this section, we examine corporate yield spreads. We initially discuss the data used. Then we discuss why yield spreads should be measured as the difference in yield to maturity on zero-coupon bonds (rather than coupon bonds) and how these rates can be estimated. Next, we examine and discuss the pattern of spreads. Finally, we compare the price of corporate bonds computed from our estimated spots with actual prices as a way of judging the reasonableness of our estimates.

A. Data

Our bond data are extracted from the Lehman Brothers Fixed Income Database distributed by Warga (1998). This database contains monthly price, accrued interest, and return data on all investment-grade corporate and government bonds. In addition, the database contains descriptive data on bonds, including coupons, ratings, and callability.

A subset of the data in the Warga database is used in this study. First, all bonds that were matrix priced rather than trader priced are eliminated from the sample.[5] Employing matrix prices might mean that all our analysis uncovers is the rule used to matrix-price bonds rather than the economic influences at work in the market. Eliminating matrix-priced bonds leaves us with a set of prices based on dealer quotes. This is the same type of data as that contained in the standard academic source of government bond data: the CRSP government bond file.[6]

Next, we eliminate all bonds with special features that would result in their being priced differently. This means we eliminate all bonds with options (e.g., callable bonds or bonds with a sinking fund), all corporate floating rate debt, bonds with an odd frequency of coupon payments, government flower bonds, and inflation-indexed government bonds.

In addition, we eliminate all bonds not included in the Lehman Brothers bond indexes, because researchers in charge of the database at Lehman Brothers indicate that the care in preparing the data was much less for bonds not included in their indexes. This results in eliminating data for all bonds with a maturity of less than one year.

[5] For actively traded bonds, dealers quote a price based on recent trades of the bond. Bonds for which a dealer did not supply a price have prices determined by a rule of thumb relating the characteristics of the bond to dealer-priced bonds. These rules of thumb tend to change very slowly over time and to not respond to changes in market conditions.

[6] The only difference in the way CRSP data is constructed and our data is constructed is that over the period of our study, CRSP uses an average of bid/ask quotes from five primary dealers called randomly by the New York Federal Reserve Board rather than a single dealer. However, comparison of a period when CRSP data came from a single dealer and also from the five dealers surveyed by the Fed showed no difference in accuracy (Sarig and Warga (1989)). Also in Section II, we show that the errors in pricing government bonds when spots are extracted from the Warga data are comparable to the errors when spots are extracted from CRSP data. Thus our data should be comparable in accuracy to the CRSP data.

Finally, we eliminate bonds where the price data or return data was problematic. This involved examining the data on bonds that had unusually high pricing errors when priced using the spot curve. Bond pricing errors were examined by filtering on errors of different sizes and a final filter rule of $5 was selected.[7] Errors of $5 or larger are unusual, and this step resulted in eliminating 2,710 bond months out of our total sample of 95,278 bond months. Examination of the bonds that are eliminated because of large differences between model prices using estimated spots and recorded prices show that large differences were caused by the following:

1. The price was radically different from both the price immediately before the large error and the price after the large error. This probably indicates a mistake in recording the data.
2. The company issuing the bonds was going through a reorganization that changed the nature of the issue (such as interest rate or seniority of claims), and this was not immediately reflected in the data shown on the tape, and thus the trader was likely to have based the price on inaccurate information about the bond's characteristics.
3. A change was occurring in the company that resulted in the rating of the company to change so that the bond was being priced as if it were in a different rating class.

B. Measuring Spreads

Most previous work on corporate spreads has defined corporate spread as the difference between the yield to maturity on a coupon-paying corporate bond (or an index of coupon-paying corporate bonds) and the yield to maturity on a coupon-paying government bond (or an index of government bonds) of the same maturity.[8] We define spread as the difference between yield to maturity on a zero-coupon corporate bond (corporate spot rate) and the yield to maturity on a zero-coupon government bond of the same maturity (government spot rate). In what follows we will use the name "spot rate" rather than the longer expression "yield to maturity on a zero-coupon bond" to refer to this rate.

The basic reason for using spots rather than yield to maturity on coupon debt is that arbitrage arguments hold with spot rates, not with yield to maturity. Because a riskless coupon-paying bond can always be expressed as

[7] The methodology used to do this is described later in this paper. We also examined $3 and $4 filters. Employing a $3 or $4 filter would have eliminated few other bonds, because there were few intermediate-size errors, and we could not find any reason for the error when we examined the few additional bonds that would be eliminated.

[8] The prices in the Warga Database are bid prices as are the bond price data reported in DRI or Bloomberg. Because the difference in the bid and ask price in the government market is less than this difference in the corporate market, using bid data would result in a spread between corporate and government bonds even if the price absent the bid/ask spread were the same. However, the difference in price is small and, when translated to spot yield differences, is negligible.

a portfolio of zeros, spot rates are the rates that must be used to discount cash flows on riskless coupon-paying debt to prevent arbitrage.[9] The same is not true for yield to maturity. In addition, the yield to maturity depends on coupon. Thus, if yield to maturity is used to define the spread, the spread will depend on the coupon of the bond that is picked. Finally, calculating spread as difference in yield to maturity on coupon-paying bonds with the same maturity means one is comparing bonds with different duration and convexity.

The disadvantage of using spots is that they need to be estimated.[10] In this paper, we use the Nelson–Siegel procedure (see Appendix A) for estimation of spots. This procedure was chosen because it performs well in comparison to other procedures.[11]

C. Empirical Spreads

The corporate spread we examine is the difference between the spot rate on corporate bonds in a particular rating class and spot rates for Treasury bonds of the same maturity. Table I presents Treasury spot rates as well as corporate spreads for our sample for the three following rating classes: AA, A, and BBB for maturities from two to ten years. AAA bonds were excluded because for most of the 10-year period studied, the number of these bonds that existed and were dealer quoted was too small to allow for accurate estimation of a term structure of spots. Corporate bonds rated below BBB were excluded because data on these bonds was not available for most of the time period we studied.[12] Initial examination of the data showed that the term structure for financials was slightly different from the term structure for industrials, and so in this section, the results for each sector are reported separately.[13] In Panel A of Table I, we have presented the average difference over our 10-year sample period, 1987 to 1996. In Panels B and C we present similar results for the first and second half of our sample period. We expect these differences to vary over time.

[9] Spot rates on promised payments may not be a perfect mechanism for pricing risky bonds because the law of one price will hold as an approximation when applied to promised payments rather than risk-adjusted expected payments. See Duffie and Singleton (1999) for a description of the conditions under which using spots to discount cash flows is consistent with no arbitrage.

[10] The choice between defining spread in terms of yield to maturity on coupon-paying bonds and spot rates is independent of whether we include matrix-priced bonds in our estimation. For example, if we use matrix-priced bonds in estimating spots we will improve estimates only to the extent that the rules for matrix pricing accurately reflect market conditions.

[11] See Nelson and Siegel (1987). For comparisons with other procedures, see Green and Odegaard (1997) and Dahlquist and Svensson (1996). We also investigated the McCulloch cubic spline procedure and found substantially similar results throughout our analysis. The Nelson and Siegel model was fit using standard Gauss–Newton nonlinear least squares methods.

[12] We use both Moody's and S&P data. To avoid confusion we will always use S&P classifications, though we will identify the sources of data. When we refer to BBB bonds as rated by Moody's, we are referring to the equivalent Moody's class, named Baa.

[13] This difference is not surprising because industrial and financial bonds differ both in their sensitivity to systematic influences and to idiosyncratic shocks that occurred over the time period.

Table I

Measured Spread from Treasury

This table reports the average spread from treasuries for AA, A, and BBB bonds in the financial and industrial sectors. For each column, spot rates were derived using standard Gauss-Newton nonlinear least square methods as described in the text. Treasuries are reported as annualized spot rates. Corporates are reported as the difference between the derived corporate spot rates and the derived treasury spot rates. The financial sector and the industrial sector are defined by the bonds contained in the Lehman Brothers' financial index and industrial index, respectively. Panel A contains the average spot rates and spreads over the entire 10-year period. Panel B contains the averages for the first five years and panel C contains the averages for the final five years.

		Financial Sector			Industrial Sector		
Maturity	Treasuries	AA	A	BBB	AA	A	BBB
Panel A: 1987–1996							
2	6.414	0.586	0.745	1.199	0.414	0.621	1.167
3	6.689	0.606	0.791	1.221	0.419	0.680	1.205
4	6.925	0.624	0.837	1.249	0.455	0.715	1.210
5	7.108	0.637	0.874	1.274	0.493	0.738	1.205
6	7.246	0.647	0.902	1.293	0.526	0.753	1.199
7	7.351	0.655	0.924	1.308	0.552	0.764	1.193
8	7.432	0.661	0.941	1.320	0.573	0.773	1.188
9	7.496	0.666	0.955	1.330	0.589	0.779	1.184
10	7.548	0.669	0.965	1.337	0.603	0.785	1.180
Panel B: 1987–1991							
2	7.562	0.705	0.907	1.541	0.436	0.707	1.312
3	7.763	0.711	0.943	1.543	0.441	0.780	1.339
4	7.934	0.736	0.997	1.570	0.504	0.824	1.347
5	8.066	0.762	1.047	1.599	0.572	0.853	1.349
6	8.165	0.783	1.086	1.624	0.629	0.872	1.348
7	8.241	0.800	1.118	1.644	0.675	0.886	1.347
8	8.299	0.813	1.142	1.659	0.711	0.897	1.346
9	8.345	0.824	1.161	1.672	0.740	0.905	1.345
10	8.382	0.833	1.177	1.682	0.764	0.912	1.344
Panel C: 1992–1996							
2	5.265	0.467	0.582	0.857	0.392	0.536	1.022
3	5.616	0.501	0.640	0.899	0.396	0.580	1.070
4	5.916	0.511	0.676	0.928	0.406	0.606	1.072
5	6.150	0.512	0.701	0.948	0.415	0.623	1.062
6	6.326	0.511	0.718	0.962	0.423	0.634	1.049
7	6.461	0.510	0.731	0.973	0.429	0.642	1.039
8	6.565	0.508	0.740	0.981	0.434	0.649	1.030
9	6.647	0.507	0.748	0.987	0.438	0.653	1.022
10	6.713	0.506	0.754	0.993	0.441	0.657	1.016

There are a number of interesting results reported in this table. Note that, in general, the corporate spread for a rating category is higher for financials than it is for industrials. For both financial and industrial bonds, the corporate spread is higher for lower-rated bonds for all spots across all maturities in both the 10-year sample and the 5-year subsamples. Bonds are priced as if the ratings capture real information. To see the persistence of this influence, Figure 1 presents the time pattern of spreads on 6-year spot payments for AA, A, and BBB industrial bonds month by month over the 10 years of our sample. Note that the curves never cross. A second aspect of interest is the relationship of corporate spread to the maturity of the spot rates. An examination of Table I shows that there is a general tendency for the spreads to increase as the maturity of the spot lengthens. However, for the 10 years from 1987 to 1996, and each 5-year subperiod, the spread on BBB industrial bonds exhibits a humped shape.

The results we find can help differentiate among the corporate debt valuation models derived from option pricing theory. The upward sloping spread curve for high-rated debt is consistent with the models of Merton (1974), Jarrow, Lando, and Turnbull (1997), Longstaff and Schwartz (1995), and Pitts and Selby (1983). It is inconsistent with the humped shape derived by Kim, Ramaswamy and Sundaresan (1987). The humped shape for BBB industrial debt is predicted by Jarrow et al. (1997) and Kim et al. (1987), and is consistent with Longstaff and Schwartz (1995) and Merton (1974) if BBB is considered low-rated debt.[14] However, one should exercise care in interpreting these results, for, as noted by Helwege and Turner (1999), the tendency of less risky companies within a rating class to issue longer-maturity debt might tend to bias yield and to some extent spots on long maturity bonds in a downward direction.

We will now examine the results of employing spot rates to estimate bond prices.

D. Fit Error

One test of our data and procedures is to see how well the spot rates extracted from coupon bond prices explain those prices. We do this by directly comparing actual prices with the model prices derived by discounting coupon and principal payments at the estimated spot rates. Model price and actual price can differ because of errors in the actual price and because bonds within the same rating class, as defined by a rating agency, are not homogenous. We calculate model prices for each bond in each rating category every month using the spot yield curves estimated for that rating class in that month. For each month, average error (error is measured as actual minus model price) and the square root of the average squared error are calculated. These are then averaged over the full 10 years and separately for the first and last 5 years for each rating category. The average error for all

[14] While the BBB industrial curve is consistent with the models that are mentioned, estimated default rates shown in Table IV are inconsistent with the assumptions these models make. Thus, the humped BBB industrial curve is inconsistent with spread being driven only by defaults.

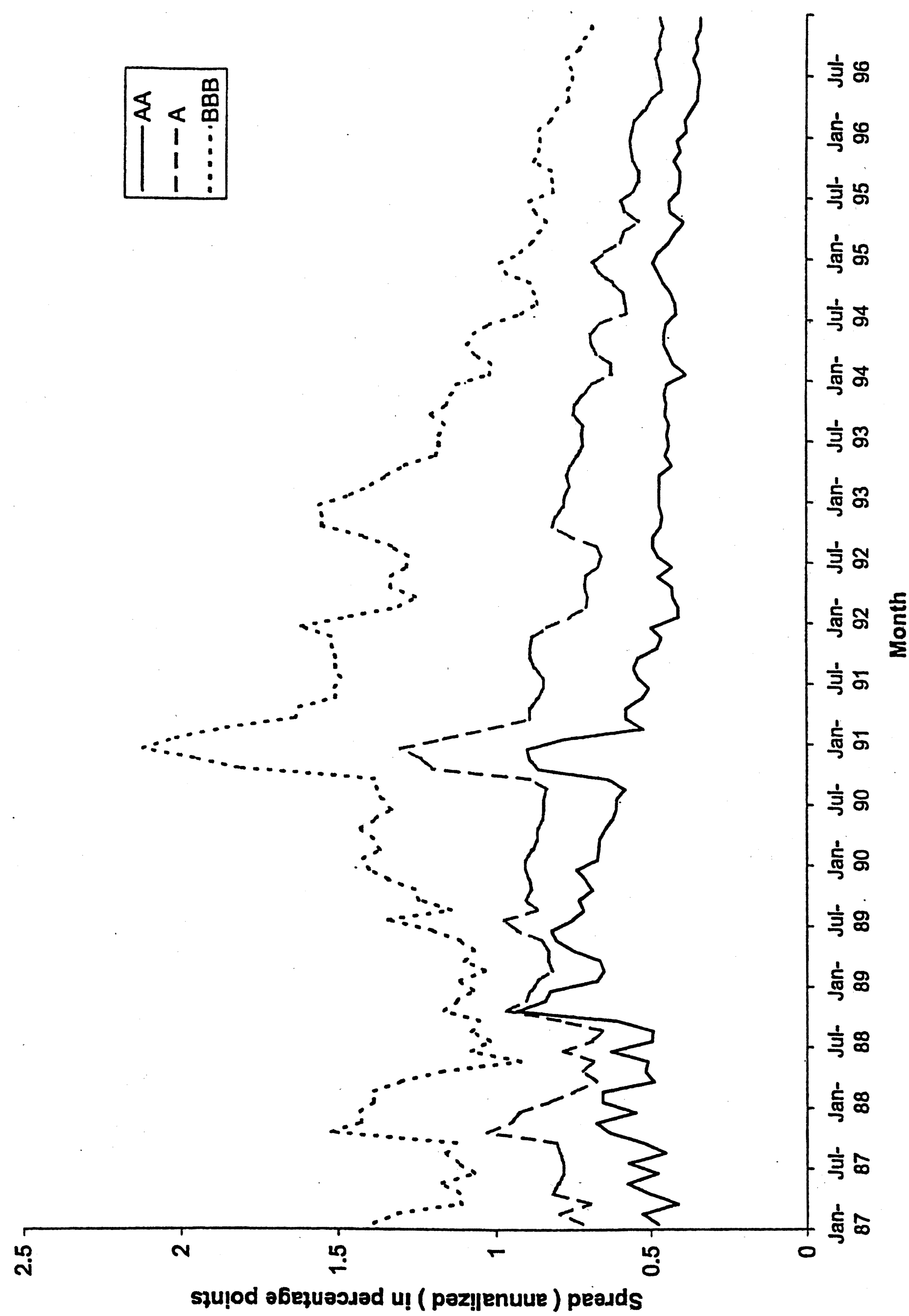


Figure 1. Empirical spreads on industrial bonds of six years maturity.

Table II

Average Root Mean Squared Errors

This table contains the average root mean squared error of the difference between theoretical prices computed from the spot rates derived from the Gauss–Newton procedure and the actual bond invoice prices. Root mean squared error is measured in cents per $100. For a given class of securities, the root mean squared error is calculated once per period. The number reported is the average of all the root mean squared errors within a class over the period indicated.

		Financial Sector			Industrial Sector		
Period	Treasuries	AA	A	BBB	AA	A	BBB
1987–1996	0.210	0.512	0.861	1.175	0.728	0.874	1.516
1987–1991	0.185	0.514	0.996	1.243	0.728	0.948	1.480
1992–1996	0.234	0.510	0.726	1.108	0.727	0.800	1.552

rating classes is very close to zero (less than one cent on a $100 bond). Root mean squared error is a measure of the variance of errors within each rating class. The average root mean squared error between actual price and estimated price is shown in Table II. The average root mean square error of 21 cents per $100 for Treasuries is comparable to the average root mean squared error found in other studies. Elton and Green (1998) had showed average absolute errors of about 16 cents per $100 using GovPX data over the period June 1991 to September 1995. GovPX data are trade prices, yet the difference in error between the studies is quite small. Green and Odegaard (1997) used the Cox, Ingersoll, and Ross (1985) procedure to estimate spot rates using data from CRSP. Although their procedure and time period are different from ours, their errors again are about the same as those we find for government bonds in our data set (our errors are smaller). The data set and procedures we are using seem to produce errors in pricing government bonds comparable in size to those found by other authors.

The average root mean squared pricing errors become larger as we examine lower grades of bonds while the average error does not change. Average root mean squared pricing errors are over twice as large for AA's as for Treasuries. The root mean squared pricing errors for BBBs are almost twice those of AAs, with the errors in As falling in between. Thus, default risk leads not only to higher spot rates, but also to greater uncertainty as to the appropriate value of the bond. This is reflected in a higher root mean squared error (variance of pricing errors). This is an added source of risk and may well be reflected in higher risk premiums, a subject we investigate shortly.[15]

[15] In a separate paper, we explore whether the difference in theoretical price and invoice price is random or related to bond characteristics. Bond characteristics do explain some of the differences but the characteristics and relationships do not change the results in this paper.

II. Estimating the Default Premium

In this section, we will estimate the magnitude of the spread that would exist under risk neutrality with the tax differences between corporates and governments ignored. Later in Section II we will introduce tax differences and examine whether expected default premium and taxes together are sufficient to explain the observed spot spread.

If investors are risk neutral, then discounting the expected cash flows from a bond at the appropriate government spot rate would produce the same value as discounting promised payments at corporate spot rates. In Appendix B, employing this insight, we show that in a risk-neutral world, the difference between corporate and government forward rates is given by

$$e^{-(r^C_{tt+1} - r^G_{tt+1})} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C}, \tag{1}$$

where C is the coupon rate; P_{t+1} is the probability of bankruptcy in period $t + 1$ conditional on no bankruptcy in an earlier period (the marginal default probabilities); a is the recovery rate assumed constant in each period; r^C_{tt+1} is the forward rate as of time 0 from t to $t + 1$ for corporate bonds; r^G_{tt+1} is the forward rate as of time 0 from t to $t + 1$ for government (risk-free) bonds; and, V_{t+1T} is the value of a T period bond at time $t + 1$ given that it has not gone bankrupt in an earlier period.

Equation (1) can be used to directly estimate the spot rate spread that would exist in a risk-neutral world between corporate and government bonds for any risk class and maturity. To perform this estimation, one needs estimates of coupons, recovery rates, and marginal default probabilities. First, the coupon was set so that a 10-year bond with that coupon would be selling close to par in all periods.[16] The only estimates available for recovery rates by rating class are computed as a function of the rating at time of issuance. Table III shows these recovery rates.[17] Estimating marginal default probabilities is more complex. Marginal default probabilities are developed from a transition matrix employing the assumption that the transition process is stationary and Markovian. We employed two separate estimates of the transition matrix, one estimated by S&P (see Altman (1997)) and one estimated by Moody's (Carty and Fons (1994)).[18] These are the two principal rating agencies for corporate debt. The transition matrixes are shown in Table IV.

[16] We examined alternative reasonable estimates for coupon rates and found only second-order effects in our results. Although this might seem inconsistent with equation (1), note that from the recursive application of equation (1) changes in C are largely offset by opposite changes in V.

[17] Recovery rates available in the literature assume that these rates are independent of the age of a bond.

[18] Each row of the transition matrix shows the probability of having a given rating in one year contingent on starting with the rating specified by the row.

Table III

Recovery Rates*

This table shows the percentage of par that a bond is worth one month after bankruptcy, given the rating shown in the first column.

Original Rating	Recovery Rate (%)
AAA	68.34
AA	59.59
A	60.63
BBB	49.42
BB	39.05
B	37.54
CCC	38.02
Default	0

*From Altman and Kishore (1998).

In year one, the marginal probability of default can be determined directly from the transition matrix and default vector, and is, for each rating class, the proportion of defaults in year one. To obtain year two defaults, we first use the transition matrix to calculate the ratings going into year two for any bond starting with a particular rating in year one. Year two defaults are then the proportion in each rating class times the probability that a bond in that class defaults by year end.[19] Table V shows the marginal default probabilities by age and initial rating class determined from the Moody's and S&P transition matrixes. The entries in this table represent the probability of default in year t given an initial rating in year 0 and given that the bond was not in default in year $t - 1$.

The marginal probability of default increases for the high-rated debt and decreases for the low-rated debt. This occurs because bonds change rating classes over time.[20] For example, a bond rated AAA by S&P has zero probability of defaulting one year later. However, given that it has not previously defaulted, the probability of it defaulting 20 years later is 0.206 percent. In the intervening years, some of the bonds originally rated AAA have migrated to lower-rated categories where there is some probability of default. At the other extreme, a bond originally rated CCC has a probability of defaulting equal to 22.052 percent in the next year, but if it survives 19 years the probability of default in the next year is only 2.928 percent. If it survives 19 years, the bond is likely to have a higher rating. Despite this drift, bonds that were rated very highly at time 0 tend to have a higher probability of staying out of default 20 years later than do bonds that initially had a low

[19] Technically, it is the last column of the squared transition matrix divided by one minus the probability of default in period 1.

[20] These default probabilities as a function of years survived are high relative to prior studies, for example, Altman (1997) and Moody's (1998).

Table IV

One One-Year Transition Probability Matrix

Panel A is taken from Carty and Fons (1994) and Panel B is from Standard and Poor's (1995). However, the category in the original references titled Non-Rated (which is primarily bonds that are bought back or issued by companies that merge) has been allocated to the other rating classes so that each row sums to one. Each entry in a row shows the probability that a bond with a rating shown in the first column ends up one year later in the category shown in the column headings.

Panel A: Moody's

	Aaa (%)	Aa (%)	A (%)	Baa (%)	Ba (%)	B (%)	Caa (%)	Default (%)
Aaa	91.897	7.385	0.718	0.000	0.000	0.000	0.000	0.000
Aa	1.131	91.264	7.091	0.308	0.206	0.000	0.000	0.000
A	0.102	2.561	91.189	5.328	0.615	0.205	0.000	0.000
Baa	0.000	0.206	5.361	87.938	5.464	0.825	0.103	0.103
Ba	0.000	0.106	0.425	4.995	85.122	7.333	0.425	1.594
B	0.000	0.109	0.109	0.543	5.972	82.193	2.172	8.903
Caa	0.000	0.437	0.437	0.873	2.511	5.895	67.795	22.052
Default	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Panel B: Standard and Poor's

	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)	Default (%)
AAA	90.788	8.291	0.716	0.102	0.102	0.000	0.000	0.000
AA	0.103	91.219	7.851	0.620	0.103	0.103	0.000	0.000
A	0.924	2.361	90.041	5.441	0.719	0.308	0.103	0.103
BBB	0.000	0.318	5.938	86.947	5.302	1.166	0.117	0.212
BB	0.000	0.110	0.659	7.692	80.549	8.791	0.989	1.209
B	0.000	0.114	0.227	0.454	6.470	82.747	4.086	5.902
CCC	0.228	0.000	0.228	1.251	2.275	12.856	60.637	22.526
Default	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

rating. However, rating migration means this does not hold for all rating classes. For example, note that after 12 years the conditional probability of default for CCCs is lower than the default probability for Bs. Why? Examining Table III shows that the odds of being upgraded to investment grade conditional on not defaulting is higher for CCC than B. Eventually, bonds that start out as CCC and continue to exist will be rated higher than those that start out as Bs. In short, the small percentage of CCC bonds that continue to exist for many years end up at higher ratings on average than the larger percentage of B bonds that continue to exist for many years.

Employing equation (1) along with the conditional default probabilities from Table V, the recovery rates from Table III, and the coupon rates estimated as explained earlier allows us to calculate the forward rates assuming risk neutrality and zero taxes. This is then converted to an estimate of the spot spread due to expected default under the same assumptions.

Table V

Evolution of Default Probability

Probability of default in year n conditional on (a) a particular starting rating and (b) not having defaulted prior to year n. These are determined using the transition matrix shown in Table IV. Panel A is based on Moody's transition matrix of Table IV, Panel A, and Panel B is based on Standard and Poor's transition matrix of Table IV, Panel B.

Panel A: Moody's

Year	Aaa (%)	Aa (%)	A (%)	Baa (%)	Ba (%)	B (%)	Caa (%)
1	0.000	0.000	0.000	0.103	1.594	8.903	22.052
2	0.000	0.004	0.034	0.274	2.143	8.664	19.906
3	0.001	0.011	0.074	0.441	2.548	8.355	17.683
4	0.002	0.022	0.121	0.598	2.842	8.003	15.489
5	0.004	0.036	0.172	0.743	3.051	7.628	13.421
6	0.008	0.053	0.225	0.874	3.193	7.246	11.554
7	0.013	0.073	0.280	0.991	3.283	6.867	9.927
8	0.019	0.095	0.336	1.095	3.331	6.498	8.553
9	0.027	0.120	0.391	1.185	3.348	6.145	7.416
10	0.036	0.146	0.445	1.264	3.340	5.810	6.491
11	0.047	0.174	0.499	1.331	3.312	5.496	5.743
12	0.060	0.204	0.550	1.387	3.271	5.203	5.141
13	0.074	0.234	0.599	1.435	3.218	4.930	4.654
14	0.089	0.265	0.646	1.474	3.157	4.678	4.258
15	0.106	0.297	0.691	1.506	3.092	4.444	3.932
16	0.124	0.329	0.733	1.532	3.022	4.229	3.662
17	0.143	0.362	0.773	1.552	2.951	4.030	3.435
18	0.163	0.394	0.810	1.567	2.878	3.846	3.241
19	0.184	0.426	0.845	1.578	2.806	3.676	3.074
20	0.206	0.457	0.877	1.585	2.735	3.519	2.928

Panel B: Standard and Poor's

Year	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)
1	0.000	0.000	0.103	0.212	1.209	5.902	22.526
2	0.002	0.017	0.154	0.350	1.754	6.253	18.649
3	0.007	0.037	0.204	0.493	2.147	6.318	15.171
4	0.013	0.061	0.254	0.632	2.424	6.220	12.285
5	0.022	0.087	0.305	0.761	2.612	6.031	10.031
6	0.032	0.115	0.355	0.879	2.733	5.795	8.339
7	0.045	0.145	0.406	0.983	2.804	5.540	7.095
8	0.059	0.177	0.457	1.075	2.836	5.280	6.182
9	0.075	0.210	0.506	1.153	2.840	5.025	5.506
10	0.093	0.243	0.554	1.221	2.822	4.780	4.993
11	0.112	0.278	0.600	1.277	2.790	4.548	4.594
12	0.132	0.313	0.644	1.325	2.746	4.330	4.272
13	0.154	0.348	0.686	1.363	2.695	4.125	4.006
14	0.176	0.383	0.726	1.395	2.639	3.934	3.780
15	0.200	0.419	0.763	1.419	2.581	3.756	3.583
16	0.225	0.453	0.797	1.439	2.520	3.591	3.408
17	0.250	0.488	0.830	1.453	2.460	3.436	3.252
18	0.276	0.521	0.860	1.464	2.400	3.292	3.109
19	0.302	0.554	0.888	1.471	2.341	3.158	2.979
20	0.329	0.586	0.913	1.475	2.284	3.033	2.860

Table VI

Mean, Minimum, and Maximum Spreads Assuming Risk Neutrality

This table shows the spread of corporate spot rates over government spot rates when taxes are assumed to be zero, and default rates and recovery rates are taken into account. The corporate forward rates are computed using equation (6). These forward rates are converted to spot rates, which are then used to compute the spreads below.

Years	AA (%)	A (%)	BBB (%)
	Panel A: Mean Spreads		
1	0.000	0.043	0.110
2	0.004	0.053	0.145
3	0.008	0.063	0.181
4	0.012	0.074	0.217
5	0.017	0.084	0.252
6	0.023	0.095	0.286
7	0.028	0.106	0.319
8	0.034	0.117	0.351
9	0.041	0.128	0.380
10	0.048	0.140	0.409
	Panel B: Minimum Spreads		
1	0.000	0.038	0.101
2	0.003	0.046	0.132
3	0.007	0.055	0.164
4	0.011	0.063	0.197
5	0.015	0.073	0.229
6	0.020	0.083	0.262
7	0.025	0.093	0.294
8	0.031	0.104	0.326
9	0.038	0.116	0.356
10	0.044	0.128	0.385
	Panel C: Maximum Spreads		
1	0.000	0.047	0.118
2	0.004	0.059	0.156
3	0.009	0.071	0.196
4	0.014	0.083	0.235
5	0.019	0.094	0.273
6	0.025	0.106	0.309
7	0.031	0.117	0.342
8	0.038	0.129	0.374
9	0.044	0.140	0.403
10	0.051	0.151	0.431

Table VI shows the zero spread due to expected default under risk-neutral valuation. The first characteristic to note is the size of the tax-free spread due to expected default relative to the empirical corporate spread discussed earlier. Our major conclusion of this section is that the zero tax spread from expected default is very small and does not account for much of the corporate spread. This can be seen numerically by comparing Tables I and VI and is illustrated graphically in Figure 2 for A-rated industrial bonds. One factor

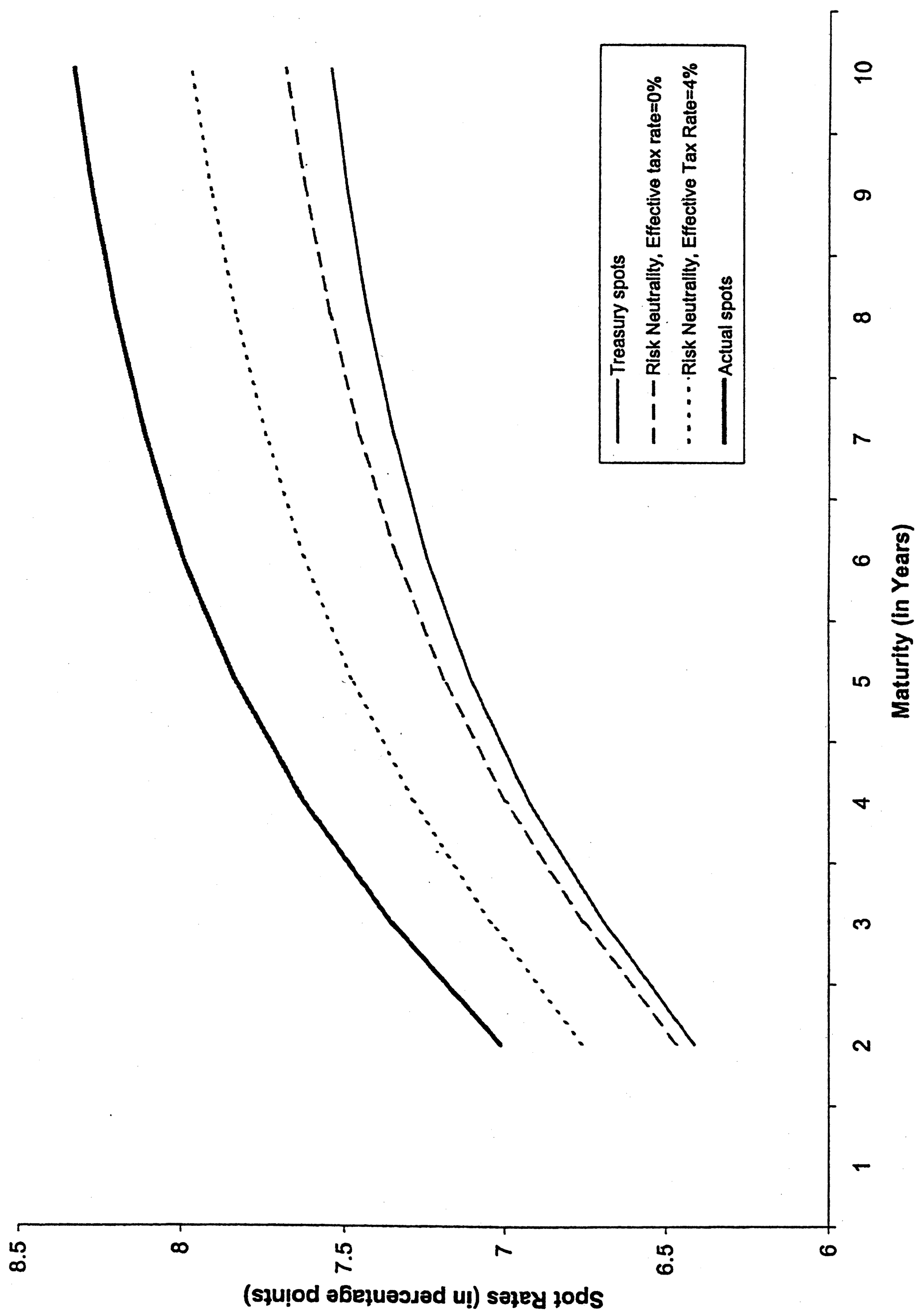


Figure 2. Spot rates for A rated industrial bonds and for treasuries.

that could cause us to underestimate the spread due to expected default is that our transition matrix estimates are not calculated over exactly the same period for which we estimate the spreads. However, there are three factors that make us believe that we have not underestimated default spreads. First, our default estimates shown in Table V are higher than those estimated in other studies. Second, the average default probabilities over the period where the transition matrix is estimated by Moody's and S&P are close to the average default probabilities in the period we estimate spreads (albeit default probabilities in the latter period are somewhat higher). Third, the S&P transition matrix that was estimated in a period with higher average default probability and that more closely matches the years in which we estimate spread results in lower estimates of defaults. However, as a further check on the effect of default rates on spreads, we calculated the standard deviation of year-to-year default rates over the 20 years ending 1996. We then increased the mean default rate by two standard deviations. This resulted in a maximum increase in spread in AA's of 0.004 percent and 0.023 percent for BBB's. Thus, even with extreme default rates, premiums due to expected losses are too small to account for the observed spreads. It also suggests that changes in premiums due to expected loss over time are too small to account for any significant part of the change in spreads over time.[21]

Also note from Table VI the zero tax spread due to default loss of AAs relative to BBBs. Although the spread for BBBs is higher, the difference in spreads because of differences in default experience is much less than the differences in the empirical corporate spreads. Differences in default rates cannot explain the differences in spreads between bonds of various rating classes. This strongly suggests that differences in spreads must be explained by other influences, such as taxes or risk premiums. The second characteristic of spreads due to expected default loss to note is the pattern of spreads as the maturity of the spot rate increases. The spread increases for longer maturity spots. This is the same pattern we observe for the empirical spreads shown in Table I. However, for AA and A the increase in premiums due to expected default loss with maturity is less than the increase in the empirical corporate spread.

III. Estimating The State Tax Premiums

Another difference between government bonds and corporate bonds is that the interest payments on corporate bonds are subject to state tax with maximum marginal rates generally between 5 and 10 percent.[22] Because state

[21] Default rates are not separately reported for industrials and financials. Thus we cannot separately calculate the size of the spread needed for default. However, recognizing that differential default rates have little impact on the spread shows that differences in the default rates for the two classes of bonds are unimportant in explaining spread differences.

[22] For a very few cities such as New York, interest income is taxable at the city level. Companies have wide latitude in determining where this interest is earned. Thus, they have the ability, in particular, to avoid taxation. Thus, the tax burden is almost exclusively at the state level and we will refer to it in this way.

tax is deductible from income for the purpose of federal tax, the burden of state tax is reduced by the federal tax rate. Nevertheless, state taxes could be a major contributor to the spreads. For example, if the coupon was 10 percent and effective state taxes were 5 percent, state taxes alone would result in a $\frac{1}{2}$ percent spread (0.05×0.10). To analyze the impact of state taxes on spreads, we introduced taxes into the analysis developed in the prior section. The derivation is contained in Appendix C. The final equation that parallels equation (1) is

$$e^{-(r^C_{tt+1} - r^G_{tt+1})} = (1 - P_{t+1}) + \frac{aP_{t+1}}{C + V_{t+1T}} - \frac{[C(1 - P_{t+1}) - (1 - a)P_{t+1}]}{C + V_{t+1T}} t_s(1 - t_g), \tag{2}$$

where t_s is the state tax rate; t_g is the federal tax rate, and other terms are as before.

The first two terms on the right-hand side are identical to the terms shown before when only default risk was taken into account. The last term is the new term that captures the effect of taxes. Taxes enter in two ways. First, the coupon is taxable and its value is reduced by taxes and is paid with probability $(1 - P_{t+1})$. Second, if the firms defaults (with probability P_{t+1}), the amount lost in default is a capital loss and taxes are recovered. Note that because state taxes are a deduction against federal taxes, the marginal impact of state taxes is $t_s(1 - t_g)$. Equation (2) is used to estimate the forward rate spread caused by the combined effects of loss due to expected default and taxes. Estimation of the forward rate spread requires, in addition to the data employed in the previous section of this paper, estimates of the term $t_s(1 - t_g)$ which we subsequently refer to as τ.

There is no direct way to measure the size of the tax terms. We employed three different procedures to measure the size of τ. The first, and the one we prefer, involves a grid search. We examine 11 different values of tax rates ranging from 0 percent to 10 percent in steps of 1 percent. For each tax rate we estimate the after-tax cash flow for each bond in every month in our sample. This was done using cash flows as defined in the multiperiod version of equation (C1) in Appendix C. Then for each month, rating class, and tax rate, we estimate the spot rates using the Nelson–Siegel procedure discussed in Appendix A, but now applied to after-tax expected cash flows. These spot yield curves are then applied to the appropriate after-tax expected cash flows to prices of all bonds in each rating class in each month. The difference between this computed price and the actual price is calculated for each tax rate. The tax rate that resulted in the smallest mean squared error between calculated price and actual price is determined, and we find that an effective tax rate of four percent results in the smallest mean squared pricing error. In addition, the four percent rate produces errors that were lower (at the five percent significance level) than any other rate except three percent. Because errors were lower on average with the four percent rate, we employ this rate for later analysis.[23]

As a reality check on the estimation of τ, we examined the tax codes in existence in each state. For most states, maximum marginal state tax rates range between 5 percent and 10 percent.[24] Because the marginal tax rate used to price bonds should be a weighted average of the active traders, we assume that a maximum marginal tax rate would be approximately the mid-point of the range of maximum state taxes, or 7.5 percent. In almost all states, state tax for financial institutions (the main holder of bonds) is paid on income subject to federal tax. Thus, if interest is subject to maximum state rates, it must also be subject to maximum federal tax, and we assume the maximum federal tax rate of 35 percent. This yields an estimate of τ of 4.875 percent.

A definite upper limit on the size of τ can be established by examining AA bonds (our highest rated category) and assuming that no risk premium exists for these bonds. If we make this assumption, the derived tax rate that explains AA spreads is 6.7 percent. There are many combinations of federal and state taxes that are consistent with this number. However, as noted above, because state tax is paid on federal income, it is illogical to assume a high state rate without a corresponding high federal rate. Thus, the only pair of rates that would explain spreads on AAs is a state tax rate of 10.3 percent and a federal rate of 35 percent. There are very few states with a 10 percent rate. Thus, it is hard to explain spreads on AA bonds with taxes and default rates. A risk premium appears to be present even for these bonds.

The corporate spreads that arise from the combined effects of expected default loss and our three tax estimates are shown in Table VII. In Table VII we have used the forward rates determined from equation (2) to calculate spot rates. Note first that the spreads in Table VII are less than those found empirically, as shown in Table I, and that, for our best estimate of effective state taxes (four percent) or for the estimate obtained from estimating rates directly, state taxes are more important than expected loss due to default in explaining spreads. This can be seen by comparing Tables VII, Panels A and B, and Table VI, or by examining Figure 2. Recall that increasing default probabilities by two standard deviations only increased the spread for AA bonds by 0.003 percent. Thus, increasing defaults to an extreme historical level plus adding on maximum or estimated tax rates are insufficient to explain the corporate spreads found empirically.

Examining Panel C of Table VII shows the spread when we apply the effective tax rate of 6.7 percent that explains AA spread to A and BBB rated bonds. Note that the tax rate that explains the spreads on AA debt underestimate the spreads on A and BBB bonds. Taxes, expected default losses,

[23] One other estimate in the literature that we are aware of is that produced by Severn and Stewart (1992), who estimate state taxes at five percent.

[24] See Commerce Clearing House (1997).

Table VII

Mean, Minimum, and Maximum Spreads with Taxes, Assuming Risk Neutrality

This table shows the spread of corporate spot rates over government spot rates when taxes as well as default rates and recovery rates are taken into account. The corporate forward rates are computed using equation (9). These forward rates are converted to spot rates, which are then used to compute the spreads below.

Years	AA (%)	A (%)	BBB (%)
Panel A: Mean Spreads with Effective Tax Rate of 4.875%			
1	0.358	0.399	0.467
2	0.362	0.410	0.501
3	0.366	0.419	0.535
4	0.370	0.429	0.568
5	0.375	0.438	0.601
6	0.379	0.448	0.632
7	0.383	0.457	0.662
8	0.388	0.466	0.691
9	0.393	0.476	0.718
10	0.398	0.486	0.744
Panel B: Mean Spreads with Effective Tax Rate of 4.0%			
1	0.292	0.334	0.402
2	0.296	0.344	0.436
3	0.301	0.354	0.470
4	0.305	0.364	0.504
5	0.309	0.374	0.537
6	0.314	0.383	0.569
7	0.319	0.393	0.600
8	0.324	0.403	0.629
9	0.329	0.413	0.657
10	0.335	0.423	0.683
Panel C: Mean Spreads with Effective Tax Rate of 6.7%			
1	0.496	0.537	0.606
2	0.501	0.547	0.639
3	0.505	0.557	0.672
4	0.508	0.566	0.704
5	0.512	0.575	0.735
6	0.516	0.583	0.765
7	0.520	0.592	0.794
8	0.524	0.600	0.821
9	0.528	0.609	0.847
10	0.532	0.618	0.871

and the risk premium inherent in AA bonds underestimate the corporate spread on lower-rated bonds. Furthermore, as shown in Table VII, Panel C, the amount of the underestimate goes up as the quality of the bonds examined goes down. The inability of tax and expected default losses to explain the corporate spread for AA's even at extreme tax rates and the inability to explain the difference in spreads between AA's and BBB's suggest a nonzero risk premium. State taxes have been ignored in almost all modeling of the spread (see, e.g., Das and Tufano (1996), Jarrow et al. (1997), and Duffee (1998)). Our results indicate that state taxes should be an important influence that should be included in such models if they are to help us understand the causes of corporate bond spreads.

IV. Risk Premiums For Systematic Risk

As shown in the last section, premiums due to expected default losses and state tax are insufficient to explain the corporate bond spread. Thus, we need to examine the unexplained spread to see if it is indeed a risk premium. There are two issues that need to be addressed. What causes a risk premium and, given the small size of the expected default loss, why is the risk premium so large?[25]

If corporate bond returns move systematically with other assets in the market whereas government bonds do not, then corporate bond expected returns would require a risk premium to compensate for the nondiversifiability of corporate bond risk, just like any other asset. The literature of financial economics provides evidence that government bond returns are not sensitive to the influences driving stock returns.[26] There are two reasons why changes in corporate spreads might be systematic. First, if expected default loss were to move with equity prices, so while stock prices rise default risk goes down and as stock prices fall default risk goes up, it would introduce a systematic factor. Second, the compensation for risk required in capital markets changes over time. If changes in the required compensation for risk affects both corporate bond and stock markets, then this would introduce a systematic influence. We believe the second reason to be the dominant influence. We shall now demonstrate that such a relationship exists and that it explains most of the spread not explained by expected default losses and taxes. We demonstrate this by relating unexplained spreads (corporate spreads less both the premium for expected default and the tax premium as determined from equation (2)) to variables that have been used as systematic risk factors in the pricing of common stocks. By studying sensitivity to these risk factors, we can estimate the size of the premium required

[25] An alternative possibility to that discussed shortly is that we might expect a large risk premium despite the low probability of default for the following reasons. Bankruptcies tend to cluster in time and institutions are highly levered, so that even with low average bankruptcy losses, there is still a significant chance of financial difficulty at an uncertain time in the future and thus there is a premium to compensate for this risk. In addition, even if the institutional bankruptcy risk is small, the consequences of the bankruptcy of an individual issue on a manager's career may be so significant as to induce decision makers to require a substantial premium.

[26] See, for example, Elton (1999).

and see if it explains the remaining part of the spread. After examining the importance of systematic risk, we shall examine whether incorporating expected defaults as a systematic factor improves our ability to explain spreads.[27]

To examine the impact of sensitivities on unexplained spreads we need to specify a return-generating model. We can write a general return-generating model as

$$R_t = a + \sum_j \beta_j f_{jt} + e_t \tag{3}$$

for each year (2–10) and each rating class, where R_t is the return during month t; β_j is the sensitivity of changes in the spread to factor j; and f_{jt} is the return on factor j during month t. The factors are each formulated as the difference in return between two portfolios (zero net investment portfolios).

As we show below, changes in the spread have a direct mathematical relationship with the difference in return between a corporate bond and a government bond. The relationship between the return on a constant maturity portfolio and the spread in spot rates is easy to derive. Thus, if either changes in spreads or the difference in returns between corporate bonds and government bonds are related to a set of factors (systematic influences), then the other must also be related to the same factors.

Let $r^c_{t,m}$ and $r^G_{t,m}$ be the spot rates on corporate and government bonds that mature m periods later, respectively. Then the price of a pure discount bond with face value equal to one dollar is

$$P^c_{t,m} = e^{-r^c_{t,m} \cdot m} \tag{4}$$

and

$$P^G_{t,m} = e^{-r^G_{t,m} \cdot m}, \tag{5}$$

and one month later the price of m period corporate and government bonds are

$$P^c_{t+1,m} = e^{-r^c_{t+1,m} \cdot m} \tag{6}$$

and

$$P^G_{t+1,m} = e^{-r^G_{t+1,m} \cdot m}. \tag{7}$$

[27] Throughout this section we will assume a four percent effective state tax rate, which is our estimate from the prior section.

Thus, the part of the return on a constant maturity m period zero-coupon bond from t to $t + 1$ due to a change in the m period spot rate is[28]

$$R^c_{t,t+1} = \ln \frac{e^{-r^c_{t+1,m} \cdot m}}{e^{-r^c_{t,m} \cdot m}} = m(r^c_{t,m} - r^c_{t+1,m}) \tag{8}$$

and

$$R^G_{t,t+1} = \ln \frac{e^{-r^G_{t+1,m} \cdot m}}{e^{-r^G_{t,m} \cdot m}} = m(r^G_{t,m} - r^G_{t+1,m}), \tag{9}$$

and the differential return between corporate and government bonds due to a change in spread is

$$R^c_{t,t+1} - R^G_{t,t+1} = -m[(r^c_{t+1,m} - r^G_{t+1,m}) - (r^c_{t,m} - r^G_{t,m})] = -m\Delta S_{t,m}, \tag{10}$$

where $\Delta S_{t,m}$ is the change in spread from time t to $t + 1$ on an m period constant maturity bond. Thus, the difference in return between corporate and government bonds due solely to a change in spread is equal to minus m times the change in spread.

Recognize that we are interested in the unexplained spread that is the difference between the corporate government spread and that part of the spread that is explained by expected default loss and taxes. Adding a superscript to note that we are dealing with that part of the spread on corporate bonds that is not explained by expected default loss and taxes, we can write the unexplained differential in returns as

$$R^{uc}_{t,t+1} - R^G_{t,t+1} = -m[(r^{uc}_{t+1,m} - r^G_{t+1,m}) - (r^{uc}_{t,m} - r^G_{t,m})] = -m\Delta S^u_{t,m}. \tag{11}$$

There are many forms of a multi-index model that we could employ to study unexplained spreads. We chose to concentrate our results on the Fama and French (1993) three-factor model because of its wide use in the literature, but we also investigated other models including the single-index model, and some of these results will be discussed in footnotes.[29] The Fama-French model employs the excess return on the market, the return on a portfolio of small stocks minus the return on a portfolio of large stocks (the SMB factor), and the return on a portfolio of high minus low book-to-market stocks (the HML factor) as its three factors.

[28] This is not the total return on holding a corporate or government bond, but rather the portion of the return due to changing spread (the term we wish to examine).

[29] We used two other multifactor models, the Connor and Korajczyk (1993) empirically derived model and the multifactor model tested by us earlier. See Elton et al. (1999). These results will be discussed in footnotes. We thank Bob Korajczyk for supplying us with the monthly returns on the Connor and Korajczyk factors.

Table VIII shows the results of regressing return of corporates over governments derived from the change in unexplained spread for industrial bonds (as in equation (5)) against the Fama–French factors.[30] The regression coefficient on the market factor is always positive and is statistically significant 20 out of 27 times. This is the sign we would expect on the basis of theory. This holds for the Fama–French market factor, and also holds (see Table VIII) for the other Fama–French factors representing size and book-to-market ratios. The return is positively related to the SMB factor and to the HML factor.[31] Notice that the sensitivity to all of these factors tends to increase as maturity increases and to increase as quality decreases. This is exactly what would be expected if we were indeed measuring risk factors. Examining financials shows similar results except that the statistical significance of the regression coefficients and the size of the R^2 is higher for AA's.

It appears that the change in spread not related to taxes or expected default losses is at least in part explained by factors that have been successful in explaining changes in returns over time in the equity market. We will now turn to examining cross-sectional differences in average unexplained premiums. If there is a risk premium for sensitivity to stock market factors, differences in sensitivities should explain differences in the unexplained premium across corporate bonds of different maturity and different rating class. We have 27 unexplained spreads for industrial bonds and 27 for financial bonds since maturities range from 2 years through 10 years, and there are three rating classifications. When we regress the average unexplained spread against sensitivities for industrial bonds, the cross-sectional R^2 adjusted for degrees of freedom is 0.32, and for financials it is 0.58. We have been able to account for almost one-third of the cross-sectional variation in unexplained premiums for industrials and one-half for financial bonds.[32]

Another way to examine this is to ask how much of the unexplained spread the sensitivities can account for. For each maturity and risk class of bonds, what is the size of the unexplained spread that existed versus the size of the estimated risk premium where the estimated premium is determined by multiplying the sensitivity of the bonds to each of the three factors times the price of each of these factors over the time period? For industrials, the average

[30] If we find no systematic influences it does not imply that the unexplained returns are not risk premiums due to systematic influences. It may simply mean that we have failed to uncover the correct systematic influences. However, finding a relationship is evidence that the unexplained returns are due to a risk premium.

[31] The results are almost identical using the Connor and Korajczyk empirically derived factors or the Elton et al. (1999) model. When a single-factor model is used, 20 out of 27 betas are significant with an of R^2 about 0.10.

[32] Employing a single index model using sensitivity to the excess return on the S&P index leads to R^2 of 0.21 and 0.43 for industrial and financial bonds, respectively. Because returns on government bonds are independent of stock factors, the beta of the change in spreads with stock excess returns is almost completely due to the effect of the stock market return on corporate bond returns. The beta for BBB industrials averages 0.26, whereas for five-year bonds, the betas ranged from 0.12 to 0.76 across rating categories. Although bond betas are smaller than stock betas, the premium to be explained is also much smaller.

Table VIII

Relationship Between Returns and Fama–French Risk Factors

This table shows the results of the regression of returns due to a change in the unexplained spread on the Fama–French risk factors, viz. (a) the market excess return (over T-bills) factor, (b) the small minus big factor, and (c) the high minus low book-to-market factor. The results reported below are for industrial corporate bonds. Similar results were obtained for bonds of financial firms. The values in parentheses are t-values.

Maturity	Constant	Market	SMB	HML	Adj-R^2
		Panel A: Industrial AA-rated Bonds			
2	−0.0046	0.0773	0.1192	−0.0250	0.0986
	−(0.297)	(2.197)	(2.318)	−(0.404)	
3	−0.0066	0.1103	0.2045	0.0518	0.0858
	−(0.286)	(2.114)	(2.680)	(0.563)	
4	−0.0058	0.1238	0.2626	0.0994	0.0846
	−(0.210)	(1.983)	(2.877)	(0.903)	
5	−0.0034	0.1260	0.3032	0.1261	0.0801
	−(0.109)	(1.791)	(2.949)	(1.018)	
6	−0.0001	0.1222	0.3348	0.1414	0.0608
	−(0.003)	(1.463)	(2.742)	(0.961)	
7	0.0035	0.1157	0.3621	0.1514	0.0374
	(0.077)	(1.116)	(2.391)	(0.829)	
8	0.0073	0.1080	0.3873	0.1586	0.0195
	(0.129)	(0.839)	(2.059)	(0.700)	
9	0.0112	0.0996	0.4119	0.1650	0.0076
	(0.163)	(0.635)	(1.798)	(0.598)	
10	0.0151	0.0912	0.4356	0.1704	−0.0002
	(0.184)	(0.489)	(1.598)	(0.519)	
		Panel B: Industrial A-rated Bonds			
2	−0.0081	0.1353	0.1831	0.0989	0.1372
	−(0.437)	(3.202)	(2.965)	(1.329)	
3	−0.0119	0.1847	0.3072	0.1803	0.2068
	−(0.534)	(3.631)	(4.134)	(2.013)	
4	−0.0123	0.2178	0.3911	0.2619	0.2493
	−(0.501)	(3.904)	(4.796)	(2.666)	
5	−0.0105	0.2419	0.4498	0.3424	0.2754
	−(0.403)	(4.068)	(5.176)	(3.270)	
6	−0.0077	0.2616	0.4952	0.4222	0.2647
	−(0.262)	(3.899)	(5.050)	(3.573)	
7	−0.0044	0.2792	0.5345	0.5014	0.226
	−(0.125)	(3.480)	(4.560)	(3.549)	
8	−0.0009	0.2958	0.5709	0.5805	0.1828
	−(0.020)	(3.032)	(4.003)	(3.378)	
9	0.0028	0.3121	0.6059	0.6596	0.1469
	(0.053)	(2.654)	(3.525)	(3.185)	
10	0.0064	0.3282	0.6407	0.7385	0.1198
	(0.105)	(2.357)	(3.149)	(3.012)	
		Panel C: Industrial BBB-rated Bonds			
2	0.0083	0.1112	0.3401	0.1259	0.0969
	(0.276)	(1.626)	(3.403)	(1.045)	
3	0.0094	0.1691	0.4656	0.2922	0.1263
	(0.255)	(2.010)	(3.787)	(1.972)	
4	0.0084	0.2379	0.5836	0.4605	0.1798
	(0.209)	(2.601)	(4.365)	(2.858)	
5	0.0062	0.3132	0.6987	0.6263	0.2585
	(0.153)	(3.406)	(5.199)	(3.867)	
6	0.0034	0.3919	0.8127	0.7901	0.3126
	(0.080)	(4.025)	(5.711)	(4.607)	
7	0.0004	0.4720	0.9260	0.9522	0.3122
	(0.008)	(4.147)	(5.567)	(4.750)	
8	−0.0028	0.5528	1.0395	1.1139	0.2807
	−(0.045)	(3.951)	(5.084)	(4.520)	
9	−0.006	0.6341	1.1529	1.2754	0.2445
	−(0.079)	(3.685)	(4.585)	(4.209)	
10	−0.0092	0.7154	1.2662	1.4370	0.2136
	−(0.101)	(3.446)	(4.173)	(3.930)	

risk premium is 0.813, whereas using the sensitivities and factor prices we would estimate it to be 0.660. For financials, the actual risk premium is 0.934, but using the estimated beta and prices, it is 0.605. In short, 85 percent of the industrial unexplained spread is accounted for by the three risk sensitivities and for financials it is 67 percent. If a single-factor model were used, the amount of the risk premium explained by the systematic risk would be reduced by more than one-third. Thus, the additional factors are important. Note that whether we use the cross-sectional explanatory power or the size of the estimate relative to the realized risk premium, we see that standard risk measures have been able to account for a high percentage of the unexplained spread.[33]

We tried one more set of tests. One possible explanation for our results is that the Fama–French factors are proxies for changes in default expectations. If this is the case, in cross section, the sensitivity of unexplained spreads to the factors may in part be picking up the market price of systematic changes in default expectations. To test this, we added several measures of changes in default risk to equation (3) as a fourth factor. We tried actual changes (perfect forecasting) and several distributed lag and lead models. None of the results were statistically significant or had consistent signs across different groups of bonds. Changes in default risk do not seem to contain any additional information about systematic risk beyond the information already captured by the Fama–French factors.

In this section we have shown that the change in unexplained spread is related to factors that are considered systematic in the stock market. Modern risk theory states that systematic risk needs to be compensated for and thus, common equity has to earn a risk premium. Changes in corporate spreads lead to changes in return on corporates and thus, returns on corporates are also systematically related to common stock factors with the same sign as common equity. If common equity receives a risk premium for this systematic risk, then corporate bonds must also earn a risk premium. We have shown that sensitivity to the factors that are used to explain risk premiums in common stocks explains between 2/3 and 85 percent of the spread in corporate and government rates that is not explained by the difference between promised and expected payments and taxes. This is strong evidence of the existence of a risk premium of a magnitude that has economic significance and provides an explanation as to why spreads on corporate bonds are so large.

V. Conclusion

In this paper we have examined the difference between spot rates on corporate and government bonds. We have shown that the spread can almost entirely be explained by three influences: the loss from expected

[33] Duffie and Singleton (1997) relate swap spreads to a series of interest rate variables. They find that the largest effect on spreads is prior shocks in this spread and changes in the spread between different rated corporate bonds.

defaults, state and local taxes which must be paid on corporate bonds but not on government bonds, and a premium required for bearing systematic risk. We supply estimates of the magnitude of each of these influences.

Several findings are of particular interest. The ratings of corporate bonds, whether provided by Moody's or Standard and Poor's, provide material information about spot rates. However, only a small part of the spread between corporate and treasuries and the difference in spreads on bonds with different ratings is explained by the expected default loss. For example, for 10-year A-rated industrials, expected loss from default accounts for only 17.8 percent of the spread.

Differential taxes are a more important influence on spreads. Taxes account for a significantly larger portion of the differential between corporate and treasuries than do expected losses. For example, for 10-year A-rated bonds, taxes accounted for 36.1 percent of the difference compared to the 17.8 percent accounted for by expected loss. State and local taxes are important because they are paid on the entire coupon of corporate bonds, not just on the difference in coupon between corporate and treasuries. Despite the importance of the state and local taxes in explaining return differentials, their impact has been ignored in almost all prior studies of corporate rates.

Even after we account for the impact of default and taxes, there still remains a large part of the differential between corporate and treasuries that remains unexplained. In the case of 10-year corporates, 46.17 percent of the difference is unexplained by taxes or expected default. We have shown that the vast majority of this difference is compensation for systematic risk and is affected by the same influences that affect systematic risks in the stock market. Making use of the Fama–French factors, we show that as much as 85 percent of that part of the spread that is not accounted for by taxes and expected default can be explained as a reward for bearing systematic risk.

In summary, we have been able to account for almost all of the differences between corporate rates and government rates. We have provided explicit estimates of the size of the these influences and we have shown that both state taxes and risk premiums are more important than the literature of financial economics has suggested.

Appendix A.
Determining Yield to Maturity on Zeros (Spot Rates)

Although there are several methods of determining spot rates given a set of bond prices, because of its simplicity and proven success in deriving spots we have adopted the methodology put forth by Nelson and Siegel (1987).

The Nelson and Siegel methodology involves fitting the following equations to all bonds in a given risk category to obtain the spot rate that is appropriate at any point in time:

$$D_t = e^{-r_t t}, \tag{A1}$$

and

$$r_t = a_0 + (a_1 + a_2)\left[\frac{1 - e^{-a_3 t}}{a_3 t}\right] - a_2 e^{-a_3 t}, \tag{A2}$$

where D_t is the present value as of time 0 for a payment that is received t periods in the future; r_t is the spot rate at time 0 for a payment to be received at time t; and a_0, a_1, a_2, and a_3 are parameters of the model.

The Nelson and Siegel procedure is used to estimate spot rates for different maturities for both Treasury bonds and for bonds within each corporate rating class for every month over the time period January 1987 through December 1996. The estimation procedure allows us, on any date, to use corporate coupon, principal payments, and prices of all bonds within the same rating class to estimate the full spot yield (discount rate) curve that best explains the prices of all bonds in that rating class on that date.[34]

Appendix B. Measuring the Default Premium in a Risk-Neutral World Without State Taxes

If investors were risk neutral (risk neutrality), the expected cash flows could be discounted at the government bond rate to obtain the value of a corporate bond. Consider a two-period bond using expected cash flows and risk neutrality. For simplicity, assume its par value at maturity is $1. We wish to determine its value at time 0 and we do so recursively by valuing it first at time 1 (as seen at time 0) and then at time 0. Its value as of time 1 when it is a one-period bond has three component parts: the value of the expected coupon to be received at period 2, the value of the expected principal to be received at period 2 if the bond goes bankrupt at period 2, and the value of the principal if the bond survives where all expectations are conditional on the bond surviving to period 1. For a bond with a face value of $1 this can be expressed as[35]

$$V_{12} = [C(1 - P_2) + aP_2 + (1 - P_2)]e^{-r_{12}^G}, \tag{B1}$$

[34] We also used the McCulloch procedure and found that numerical results were similar and all of the conclusions of this paper were unchanged.

[35] The assumption of receiving a constant proportion of face value has been made in the literature by Duffie (1998). We are assuming that default payment occurs at the time of default. This is consistent with the evidence that default occurs because of an inability to meet a payment. We also assume that recovery rate is a percentage of par. This is how all data is collected (e.g., Altman (1997)).

where C is the coupon rate; P_t is the probability of bankruptcy in period t conditional on no bankruptcy in an earlier period; a is the recovery rate assumed constant in each period; r^G_{tt+1} is the forward rate as of time 0 from t to $t + 1$ for government (risk-free) bonds;[36] and V_{tT} is the value of a T period bond at time t given that it has not gone bankrupt in an earlier period. Alternatively, we can value the bond using promised cash flows, according to

$$V_{12} = (C + 1)e^{-r^C_{12}}, \tag{B2}$$

where r^C_{tt+1} is the forward rate from t to $t + 1$ for corporate bonds.

Equating the two values and rearranging to solve for the difference between corporate and government forward rates, we have

$$e^{-(r^C_{12}-r^G_{12})} = (1 - P_2) + \frac{aP_2}{(1 + C)}. \tag{B3}$$

At time 0, the value of the two-period bond using risk neutral valuation is

$$V_{02} = [C(1 - P_1) + aP_1 + (1 - P_1)V_{12}]e^{-r^G_{01}} \tag{B4}$$

and using promised cash flows, its value is

$$V_{02} = [C + V_{12}]e^{-r^C_{01}}. \tag{B5}$$

Equating these expressions for V_{02} and solving for the difference in one-period spot (or forward) rates, we have

$$e^{-(r^C_{01}-r^G_{01})} = (1 - P_1) + \frac{aP_1}{V_{12} + C}. \tag{B6}$$

In general, in period t the difference in forward rates is[37]

$$e^{-(r^C_{tt+1}-r^G_{tt+1})} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C}, \tag{B7}$$

where $V_{TT} = 1$.

[36] We discount at the forward rate because this is the rate which can be contracted upon at time 0 for moving money across time.

[37] The difference in forward rates may vary across bonds with different coupons, even for bonds of the same rating class because, as discussed earlier, arbitrage on promised payments is an approximation that holds exactly only under certain assumptions (see Duffie and Singleton (1999)). If these assumptions do not hold, the estimates of spot rates obtained empirically are averages across bonds with different coupons and one single spot rate would not hold exactly for all bonds. Nevertheless, even in this case, given the size of the pricing error found in the previous section, assuming one rate is a good approximation.

Appendix C. Estimating the Impact of State Taxes

To analyze the impact of state taxes on spreads, we introduced the taxes into the analysis developed in Section II. For a one-period bond maturing at \$1, the basic valuation equation after state taxes is

$$V_{01} = [C(1-P_1)(1-t_s(1-t_g)) + aP_1 + (1-a)P_1(t_s(1-t_g)) + (1-P_1)]e^{-r_{01}^G}, \tag{C1}$$

where t_s is the state tax rate, t_g is the federal tax rate, and other terms are as before.

Equation (C1) has two terms that differ from those when taxes are not present. The change in the first term represents the payment of taxes on the coupon. The new third term is the tax refund due to a capital loss if the bond defaults.

The valuation on promised cash flows is

$$V_{01} = [C+1]e^{-r_{01}^C}, \tag{C2}$$

Equating the two expressions for V_{01} and solving for the difference between corporate and government rates, we have

$$e^{-(r_{01}^C - r_{01}^G)} = (1-P_1) + \frac{aP_1}{1+C} - \frac{[C(1-P_1)-(1-a)P_1]}{1+C}(t_s)(1-t_g). \tag{C3}$$

As in Appendix B, these equations can be generalized to the T period case. The final equation is

$$(1-P_{t+1}) + \frac{aP_{t+1}}{C+V_{t+1T}} - \frac{[C(1-P_{t+1})-(1-a)P_{t+1}]}{C+V_{t+1T}} t_s(1-t_g) = e^{-(r_{tt+1}^C - r_{tt+1}^G)}. \tag{C4}$$

This equation is used to estimate the forward rate spread because of loss due to expected default and taxes.

REFERENCES

Altman, Edward, 1989, Measuring corporate bond mortality and performance, *Journal of Finance* 44, 909–922.

Altman, Edward I., and Vellore M. Kishore, 1998, Defaults and returns on high yield bonds: Analysis through 1997, NYU Salomon Center working paper.

Altman, Edward, 1997, Rating migration of corporate bonds: Comparative results and investor/lender implications, NYU Salomon Center Working Paper Series S-97-3.

Blume, M.E., D.B. Keim, and S.A. Patel, 1991, Returns and the volatility of low grade bonds, *Journal of Finance* 46, 49–74.

Bodie, Zvi, Alex Kane, and Alan Marcus, 1993, *Investments* (Irwin, Homewood, IL).

Carty, L., and J. Fons, 1994, Measuring changes in credit quality, *Journal of Fixed Income* 4, 27–41.

Commerce Clearing House, 1997, *State Tax Handbook.* (Commerce Clearing House, Inc., Chicago, IL).

Connor, G., and R.A. Korajczyk, 1993, A test for the number of factors in an approximate factor model, *Journal of Finance* 48, 1263–1291.

Cornell, B., and R. Green, 1991, Returns and volatility of low-grade bonds, 1977–1989, *Journal of Finance* 46, 29–48.

Cox, John, John Indgersoll, and Steve Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385–408.

Cumby, Bob, and Martin Evans, 1995, The term structure of credit risk: Estimates and specification tests, NYU Salomon Center Working paper.

Dahlquist, M., and L.E.O. Svensson, 1996, Estimating the term structure of interest rates for monetary policy analysis, *Scandinavian Journal of Economics* 98, 163–183.

Das, Sanjiv R., and Peter Tufano, 1996, Pricing credit sensitive debt when interest rates, credit ratings and credit spreads are stochastic, *Journal of Financial Engineering* 5, 161–198.

Duffee, G., 1998, The relation between treasury yields and corporate bond yield spreads, *Journal of Finance* 53, 2225–2240.

Duffie, D., and K. Singleton, 1997, An econometric model of the term structure of interest rate swap yields, *Journal of Finance* 52, 1287–1322.

Duffie, D., and K. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687–720.

Elton, E., M. Gruber, and C. Blake, 1999, Common factors in active and passive portfolios, *European Finance Review* 3, 53–78.

Elton, Edwin J., 1999, Expected return, realized return, and asset pricing tests, *Journal of Finance* 54, 1199–1221.

Elton, Edwin J., and T. Clifton Green, 1998, Tax and liquidity effects in pricing government bonds, *Journal of Finance* 53, 1533–1562.

Fama, E., and K. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–57.

Fons, Jerome S., 1994, Using default rates to model the term structure of credit risk, *Financial Analysts Journal* 50, 25–32.

Goodman, L.S., 1990, High yield default rates: Is there cause for concern? *Journal of Portfolio Management* 16, 54–59.

Green, C., and A. Odegaard, 1997, Tax effects in relative pricing of U.S. government bonds, *Journal of Finance* 52, 609-633.

Helwege, J.,and C. Turner, 1999, The slope of the credit yield curve for speculative grade issues, *Journal of Finance* 54, 1869–1884.

Jarrow, R.A., D. Lando, and S. Turnbull, 1997, A Markov model for the term structure of credit risk spreads, *Review of Financial Studies* 10, 481–523.

Kim, I., K. Ramaswamy, and S. Sundaresan, 1987, The valuation of corporate fixed income securities, Manuscript, Columbia University.

Longstaff, F.A., and E.S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789–819.

Merton, R.C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Moody's Investors Service, 1998, *Historical default rates of corporate bond issuers 1920–1997* (Moody's Investors Service, New York).

Nelson, R., and F. Siegel, 1987, Parsimonious modeling of yield curves, *Journal of Business* 60, 473–489.

Pitts, C.G.C., and M.J.P. Selby, 1983, The pricing of corporate debt: a further note, *Journal of Finance* 38, 1311–1313.

Sarig, O., and A. Warga, 1989, Bond price data and bond market liquidity, *Journal of Financial and Quantitative Analysis* 24, 367–378.

Severn, A.K., and W.J. Stewart, 1992, The corporate-treasury yield spread and state taxes, *Journal of Economics and Business* 44, 161–166.

Warga, A., 1998, Fixed income data base, University of Houston, Houston, Texas.